Unbinned halo-independent methods for emerging dark matter signals

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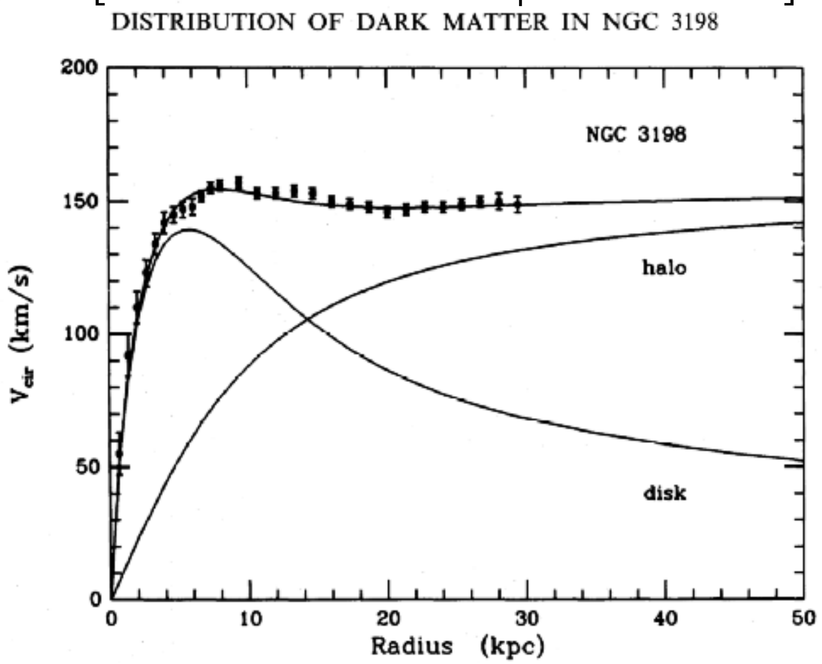
Yoni Kahn, MIT

with Matthew McCullough and Patrick Fox

1403.6830, 1409.xxxx

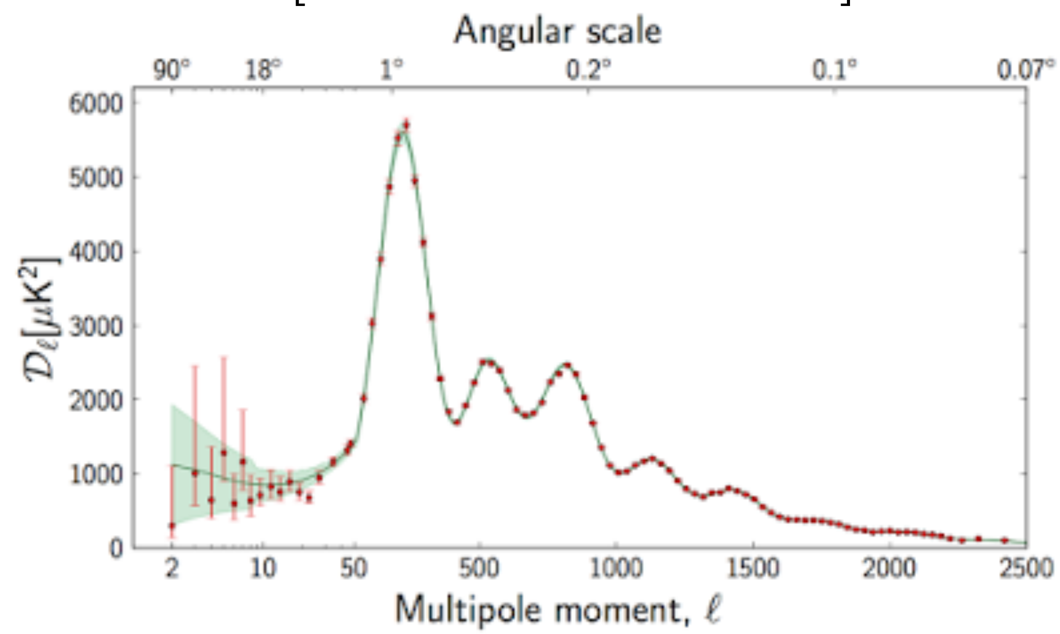
Dark matter: things we know

[van Albada et al. ApJ 295 1985]



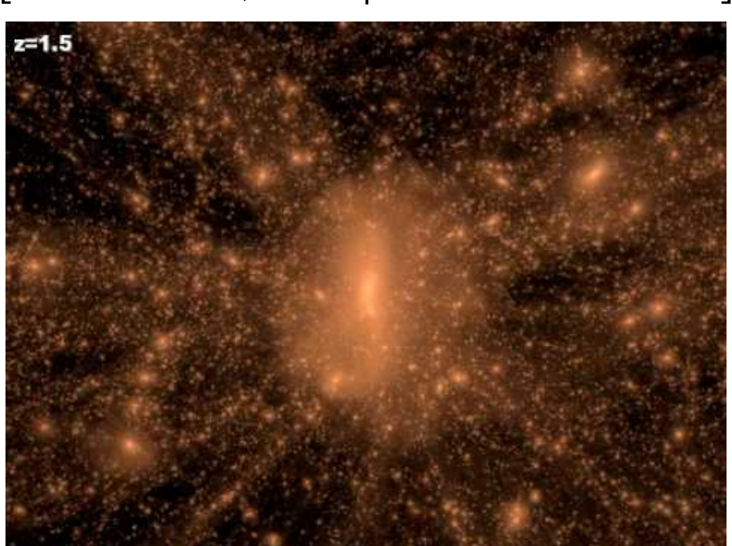
Galaxies have halos

[Planck collab. A&A 2014]



Universe is 26.8% DM

[Via Lactea, Zemp MPLA 24 2009]



DM forms structures

[Markevitch et al. ApJ 606 2003]

$\sigma/m < 1.3\,\mathrm{barn/GeV}$

Dark matter: things we don't know

- Mass? Well-motivated candidates from sub-sub-eV (axions) to 10^{13} GeV (WIMPZILLAS)
- Non-gravitational interactions with visible matter? elastic/inelastic, spin-independent/dependent, light/heavy mediator…or maybe none at all (gravitino)?
- Dark sector? SM is complex, why not a whole dark sector with multiple states, self-interactions, decays, etc?
- What does our local halo look like? Local density? Velocity distribution might be Maxwell-Boltzmann, but N-body simulations suggest not [Kuhlen et al. 0912.2358] and this has a strong effect on direct detection

How far can we get making as few assumptions as possible?

(SI)-Direct detection

$$\boxed{\frac{dR}{dE_R} = \frac{\rho_\chi \sigma_n}{2 m_\chi \mu_{n\chi}^2} N_A m_n C_T^2(A,Z) \int dE_R' G(E_R, E_R') \epsilon(E_R') F^2(E_R') \overbrace{\int_{v_{min}(E_R')}^{\infty} \frac{f(\mathbf{v}+\mathbf{v}_E)}{v} d^3v}^{= g(v_{min})}}$$

DM model | detector properties | nuclear physics | DM halo model

$v_{min}(E_R')$: min. DM velocity required for nuclear recoil E_R'

Usual method: DM model + halo model $\rightarrow$ limits/preferred values in $m_\chi - \sigma_n$ space

Halo-independent: DM model $\rightarrow$ limits/preferred values in $v_{min} - g(v_{min})$ space

No assumptions about DM halo, easy to compare multiple experiments (esp. signal vs. exclusion)

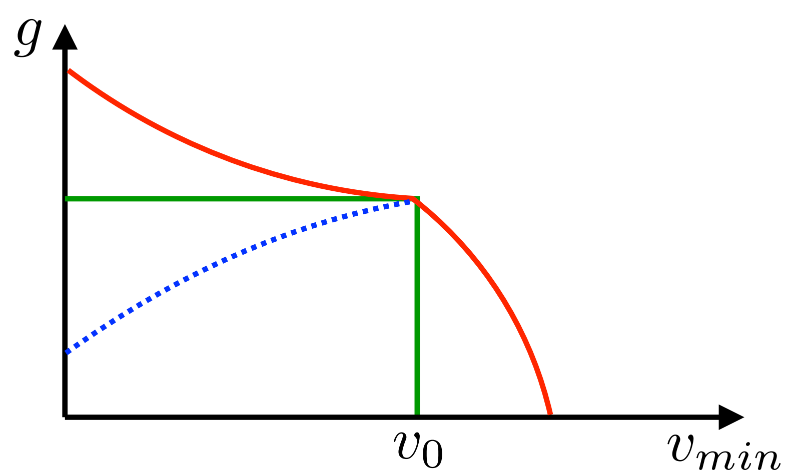
What will an emerging direct detection signal look like?

- Backgrounds are extremely low
- Excellent energy resolution
- Given current hints, probably O(1-10) events at first
- Want to keep all possible information - avoid binning in E_R-space
- Test signal against as many DM kinematics as possible (elastic, inelastic, exothermic, …)

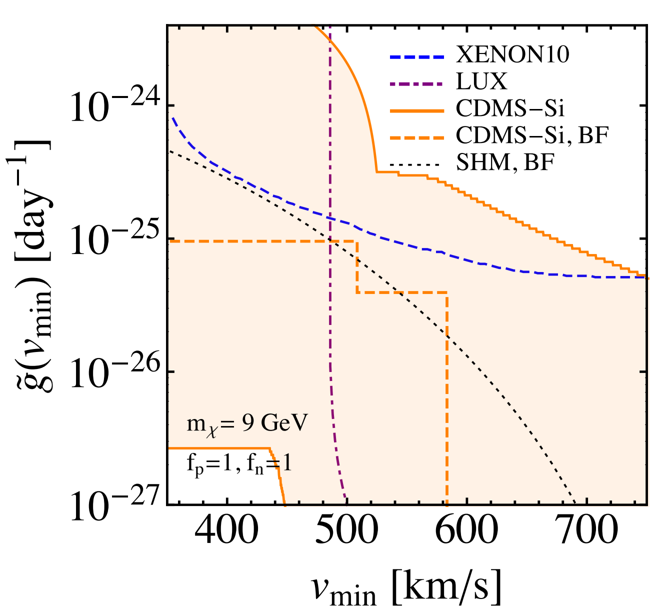
This talk: focus on unbinned methods for general kinematics

Outline

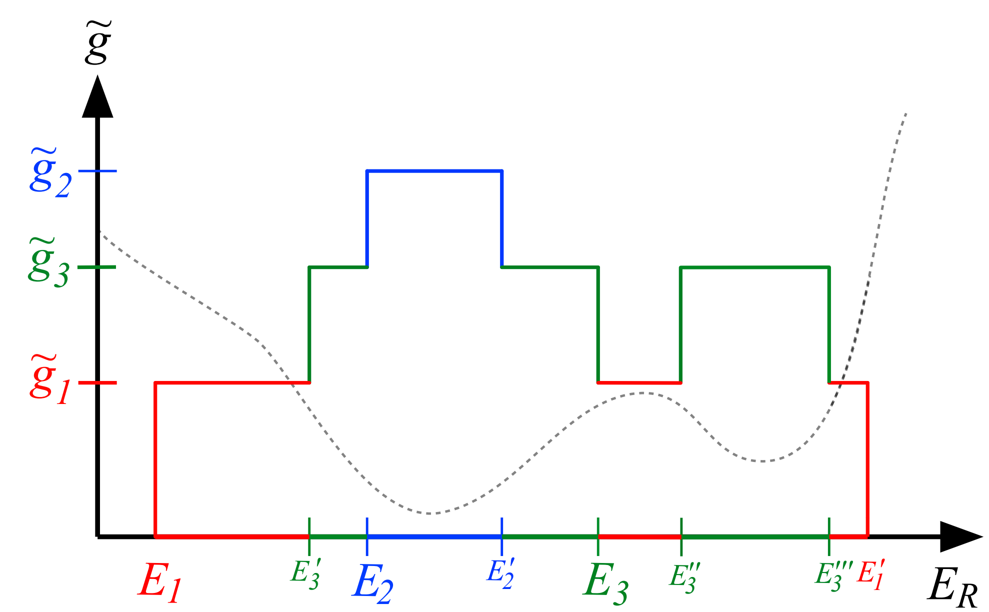
I. Exploiting monotonicity of DM velocity integral



II. Unbinned halo-independent methods for elastic scattering



III. Methods for non-vanilla DM (inelastic, exothermic, multiple channels)



I. Exploiting monotonicity

Properties of velocity integral

[Fox, Liu, Weiner 1011.1915]

Integrand is positive-definite:

$$g(v_{min}) = \int_{v_{min}}^{\infty} \frac{f(\mathbf{v} + \mathbf{v}_E)}{v} d^3v$$

Monotonically decreasing for any *f*

$\Longrightarrow$ powerful consistency conditions

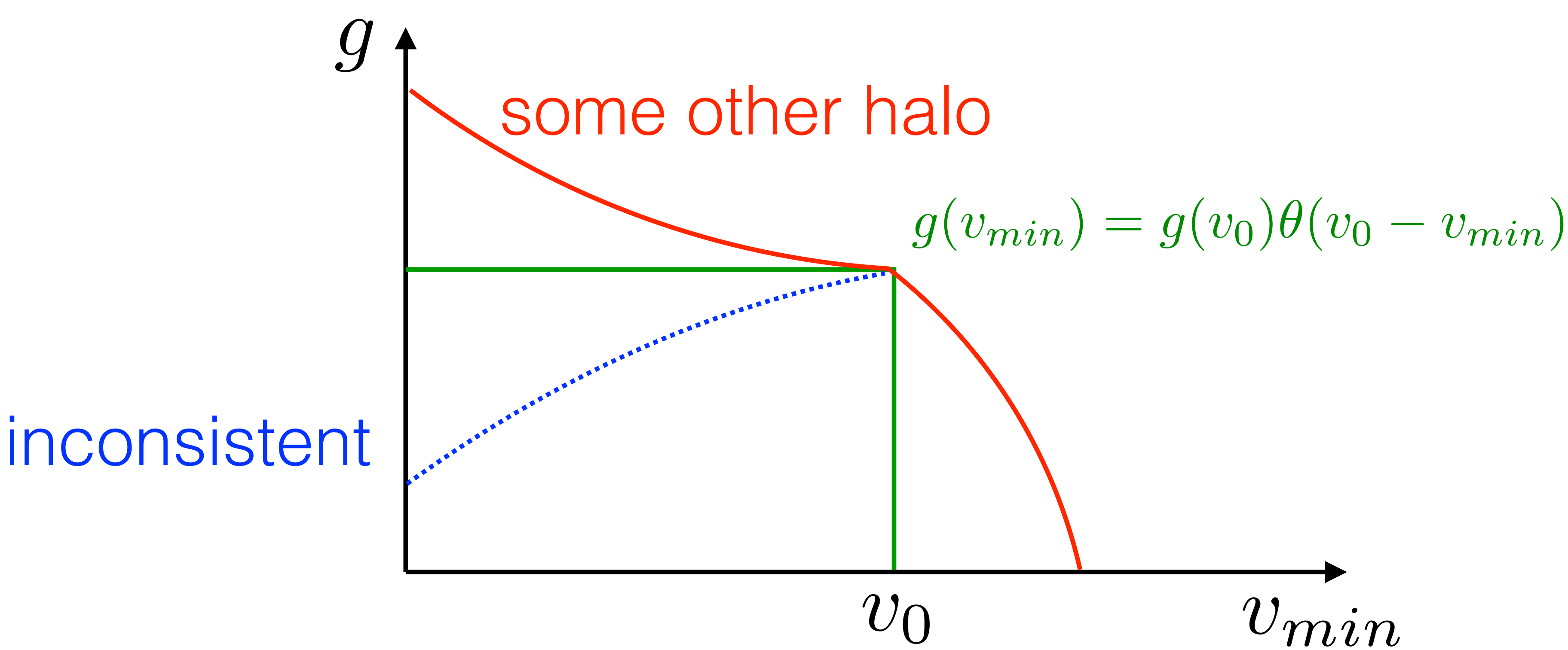
Additional kinematic input: $v_{min}(E_R)$

E.g. elastic scattering: $v_{min}(E_R) = \sqrt{\frac{m_N E_R}{2\mu_{N\chi}^2}}$

(assume elastic for this part of the talk)

Null results - setting limits

1-to-1 mapping: $E_0 \leftrightarrow v_0$



At each v_0, most conservative halo integral is step function (all others give more total events by monotonicity)

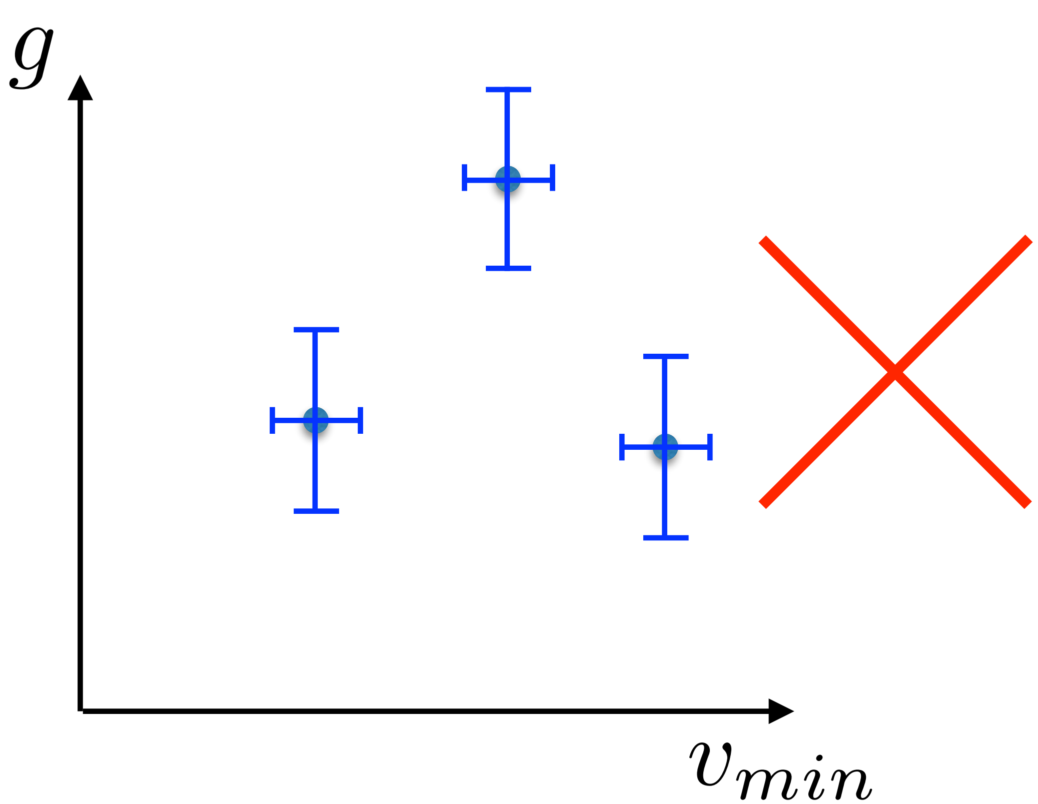
Plug into rate formula, use your favorite confidence estimator to get height $g(v_0)$, repeat for all v_0 in range

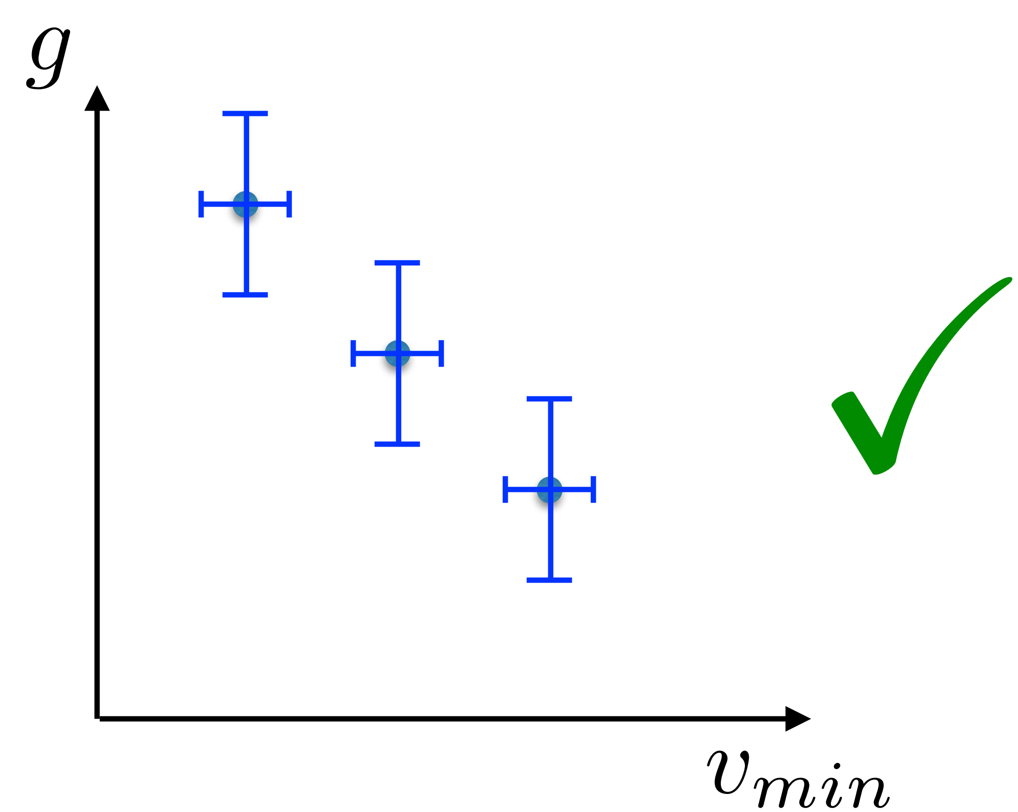
Positive signals

1-to-1 mapping for each energy bin: $[E_1, E_2] \leftrightarrow [v_1, v_2]$

Events in bin $[E_1, E_2]$ $\rightarrow$ preferred values in $g - v_{min}$ space

Can rule out a DM interpretation if preferred values are not monotonic within some confidence interval



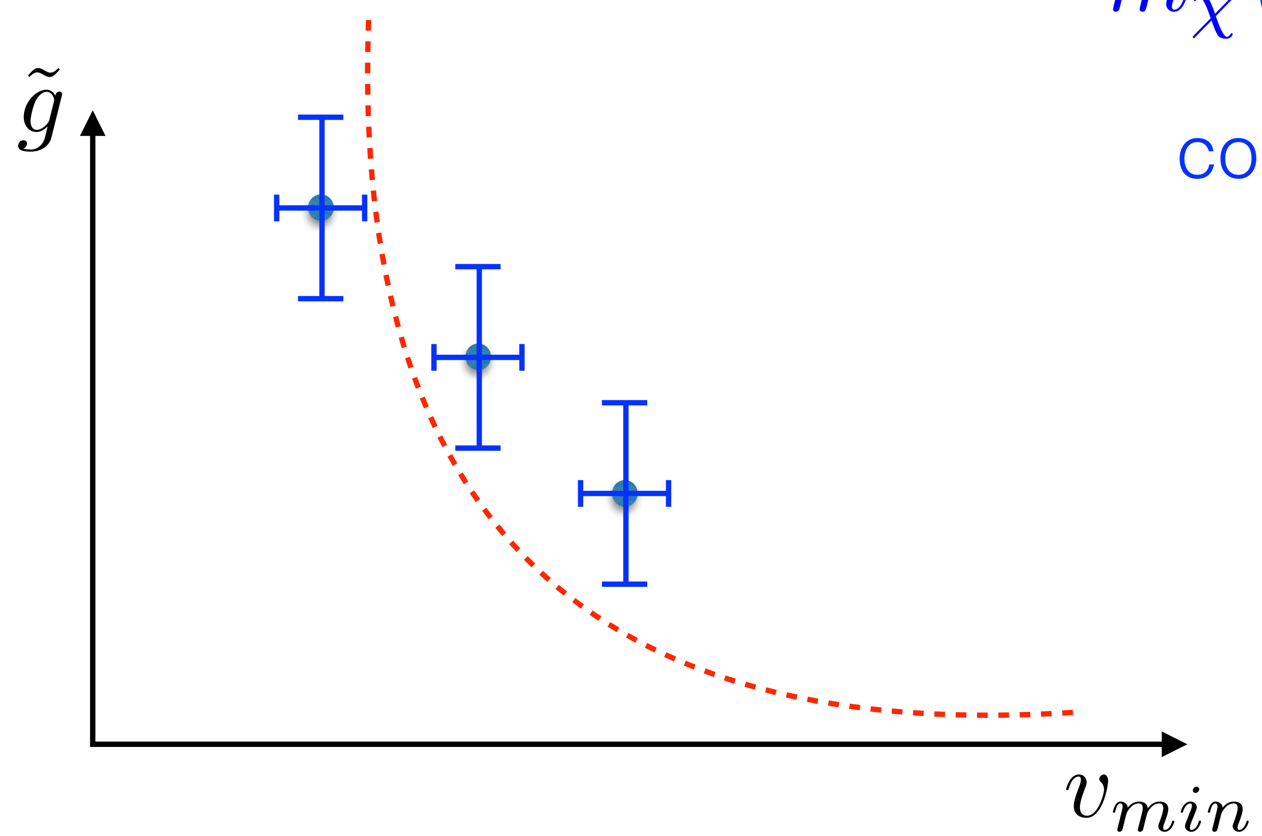


Comparing experiments

[Frandsen et al. 1111.0292]

Rescale halo integral: $\tilde{g}(v_{min}) = \frac{\rho_\chi \sigma_n}{m_\chi} g(v_{min})$

common to all expts.



Excluded = excluded for all halos

$$v_{min}(E_R) = \sqrt{\frac{m_N E_R}{2\mu_{N\chi}^2}}$$

can even compare non-overlapping E_R ranges depending on m_N

Nicely complementary to usual $m_\chi - \sigma_n$ presentation

II. Unbinned halo-independent methods for elastic scattering

Positive signals - unbinned

[YK, Fox, McCullough, 1403.6830]

Binning best for lots of events, but ambiguous and not so useful for emerging signals with few events

Avoid this by using extended maximum likelihood method:

[R.J. Barlow, Nucl. Instrum. Meth. A297 (1990)]

total expected events for given params.

total rate (signal + background)

$$\mathcal{L} = \frac{e^{-N_E}}{N_O!} \prod_{i=1}^{N_O} \frac{dR_T}{dE_R}\bigg|_{E_R=E_i}$$

observed recoil energies (no binning!)

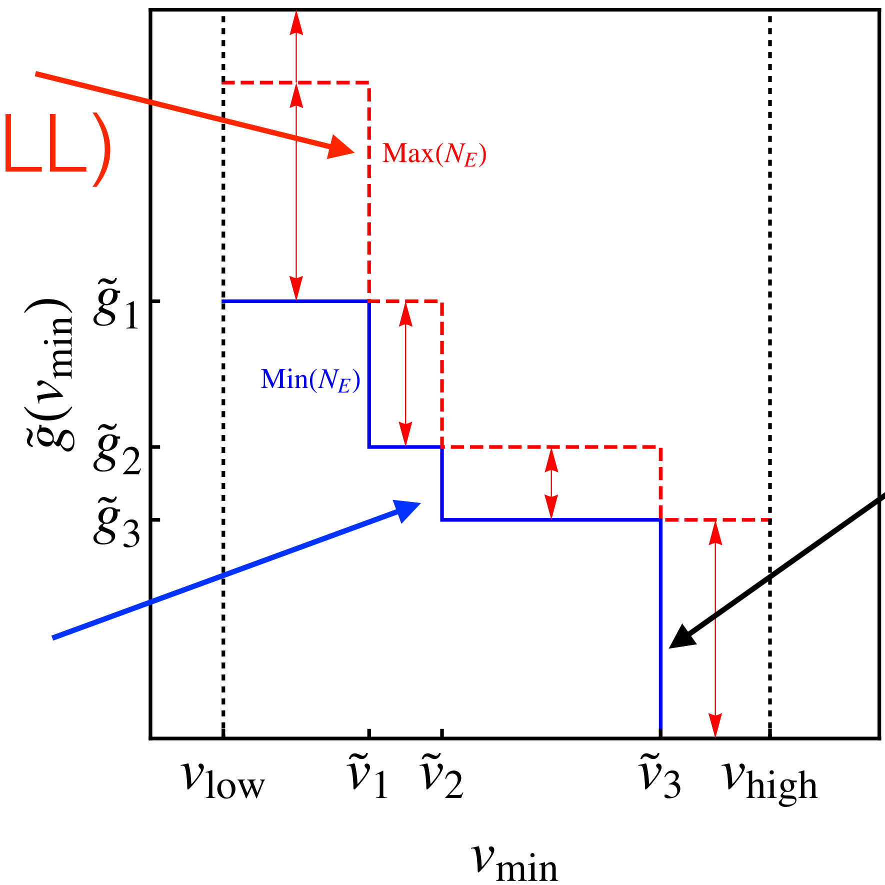
Key point: $\mathcal{L}$ penalizes against more expected events

Positive signals - unbinned

$$N_E \propto \int dE'_R \, \tilde{g}(v_{min}(E'_R)) \propto \int dv_{min} \, \tilde{g}(v_{min})$$

Monotonicity prefers step function form for $\tilde{g}(v_{min})$

worst fit
(for extended LL)



best fit

step positions fixed by measured recoil energies (for perfect energy resolution)

$$\tilde{v}_i = v_{min}(E_i)$$

Finite-resolution effects

$$\left.\frac{dR_T}{dE_R}\right|_{E_i} = \left.\frac{dR_{BG}}{dE_R}\right|_{E_i} + \frac{N_A m_n}{2\mu_{n\chi}^2} C_T^2(A,Z) \int dE'_R G(E_i, E'_R) \epsilon(E'_R) F^2(E'_R) \tilde{g}(v_{min}(E'_R))$$

detector resolution function

$\tilde{g}$ contributes for all E'_R, not just measured E_i - does step function still maximize likelihood?

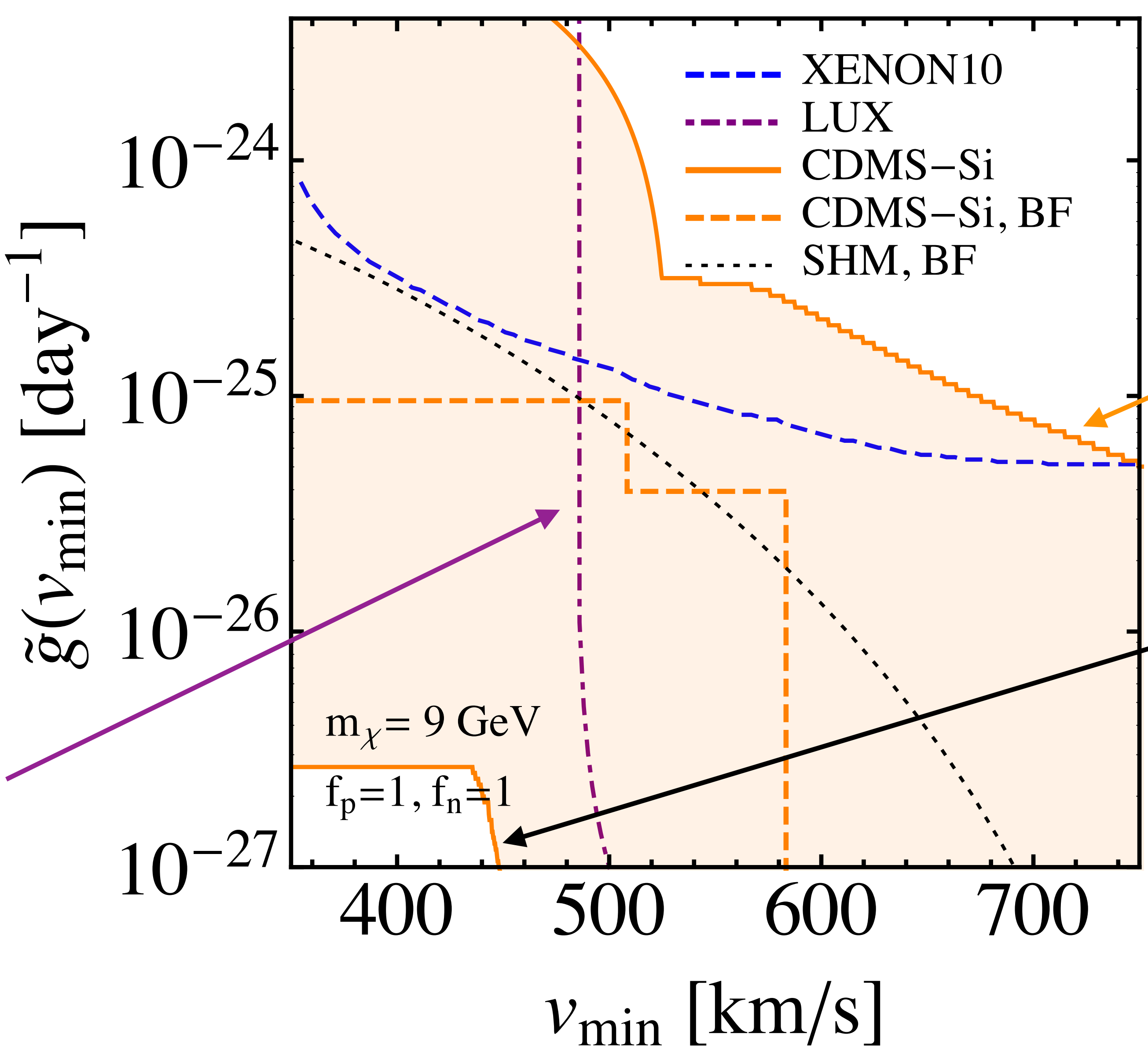
Yes, but positions of steps can float: no longer fixed at $v_{min}(E_i)$

(proof by variational techniques)

$\Longrightarrow$ $2N_O$-parameter maximization (heights and positions)

Example: CDMS-Si vs. LUX and XENON-10

Apply method for 3 observed CDMS events vs. null results of LUX and XENON



envelope of preferred values for χ^2 w/5 d.o.f.

if bottom of envelope is excluded, DM interpretation ruled out for all halos

LUX excludes both standard halo model and best-fit halo

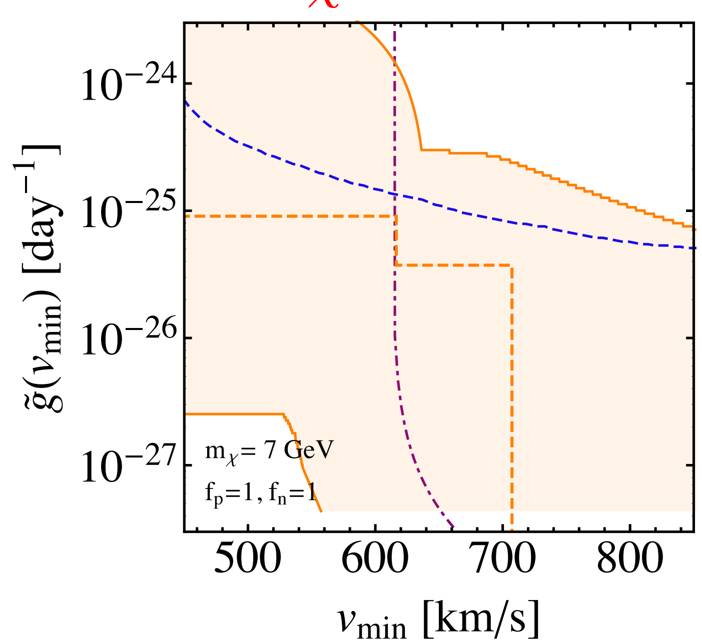
One plot, all DM masses

$$m_\chi \to m'_\chi : \quad v'_{min}(E_R) = \frac{\mu_{N\chi}}{\mu_{N\chi'}} v_{min}(E_R), \qquad \tilde{g}' = \frac{\mu^2_{n\chi'}}{\mu^2_{n\chi}} \tilde{g}$$

1-to-1 mapping, preserves order of events

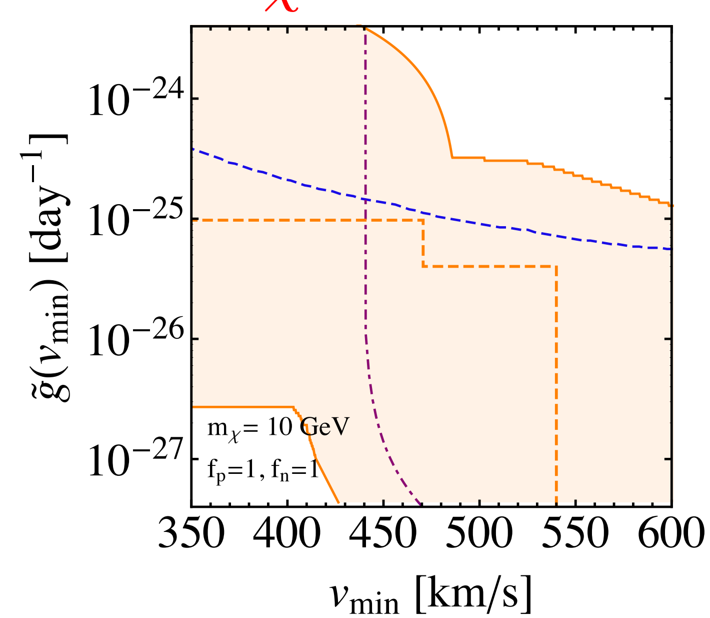
rescaling each step

$m_\chi = 7$ GeV



One plot contains all info necessary to compare expts. for any DM mass!*

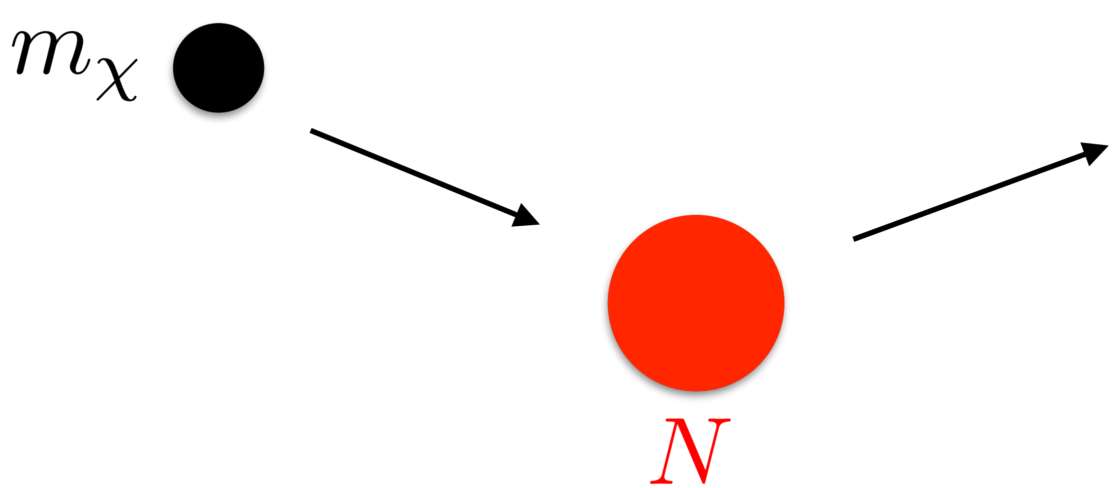
$m_\chi = 10$ GeV



*for single target material

III. Generalization for non-vanilla DM

Inelastic kinematics



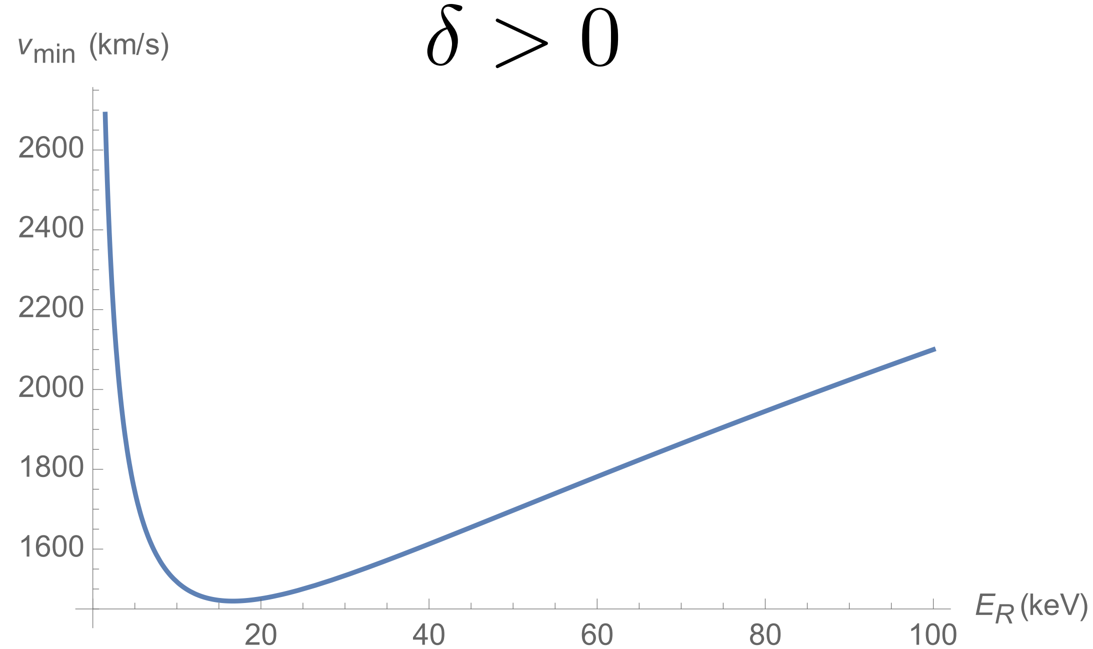
$m'_\chi = m_\chi + \delta/2$

$\delta > 0$: “inelastic”

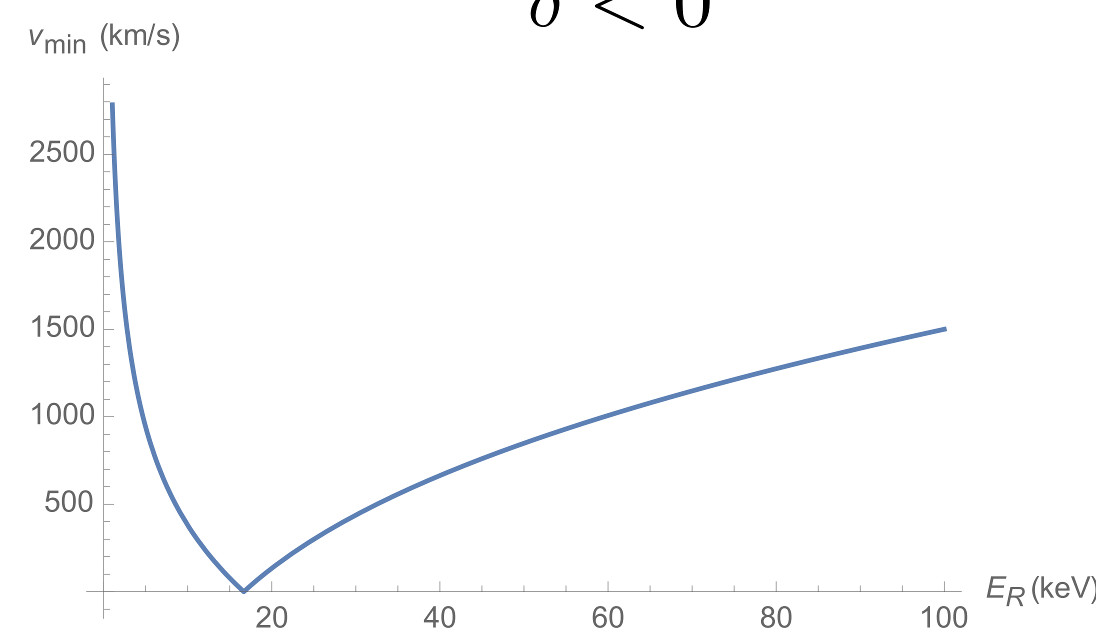
$\delta < 0$: “exothermic”

$$v_{min}(E_R) = \frac{1}{\sqrt{2 m_N E_R}} \left| \frac{m_N E_R}{\mu_{N\chi}} - \delta \right|$$

$\delta > 0$

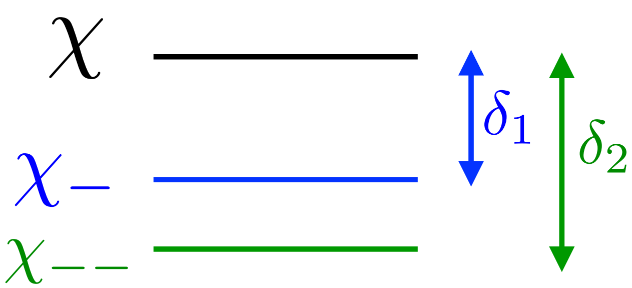


$\delta < 0$



Mapping $E_R \leftrightarrow v_{min}$ no longer 1-to-1!

Multi-channel scattering kinematics

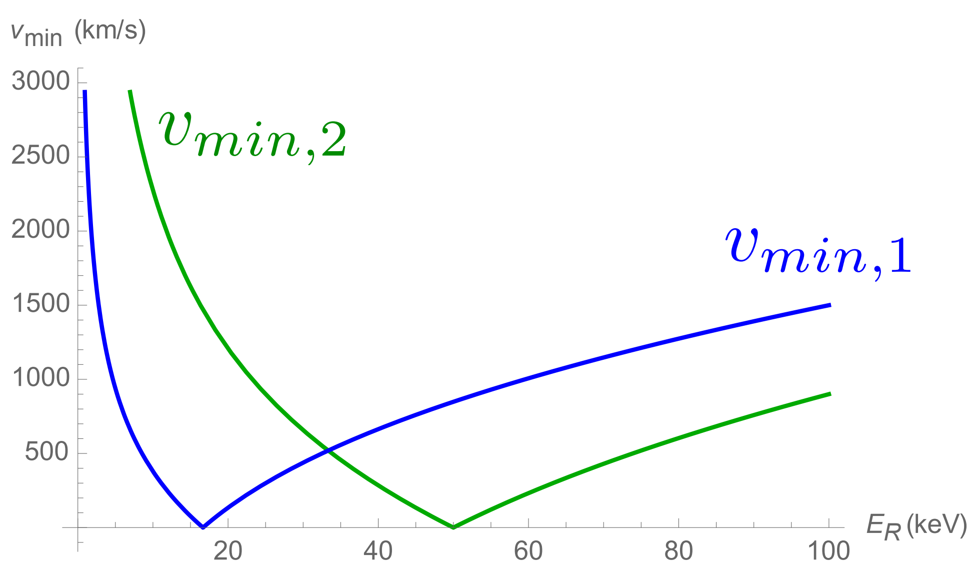


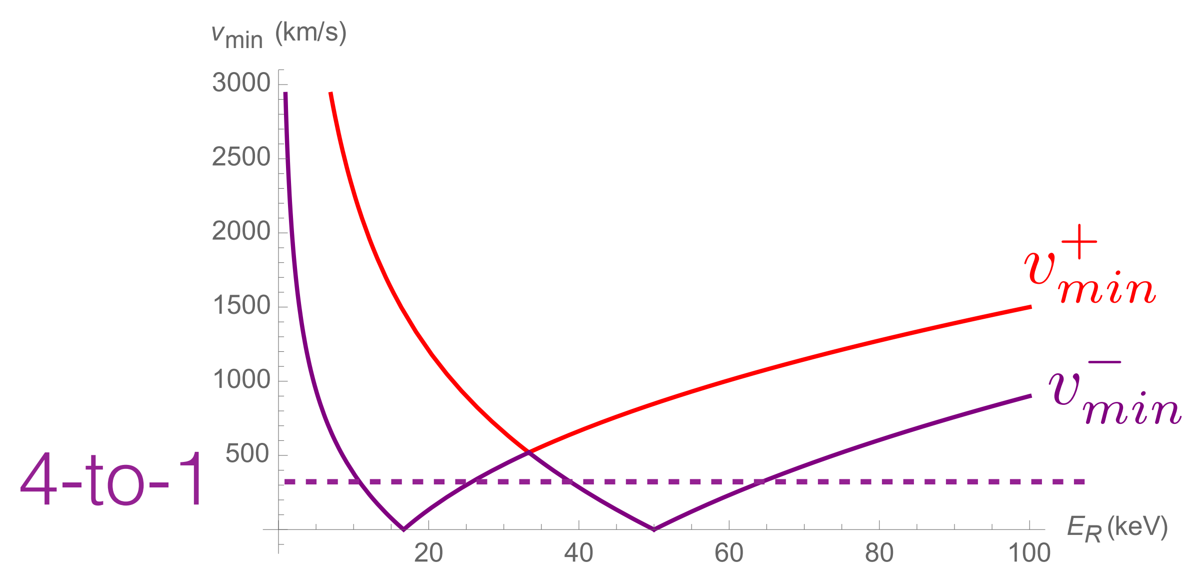
Model-independent parameterization:

$$\sigma_{\chi N \to \chi_- N} = \alpha_1 \sigma_n \qquad \alpha_1 + \alpha_2 = 1$$
$$\sigma_{\chi N \to \chi_{--} N} = \alpha_2 \sigma_n$$

$$\frac{dR_{tot}}{dE_R} \propto \alpha_1 \tilde{g}(v_{min,1}) + \alpha_2 \tilde{g}(v_{min,2})$$

positivity of integrand $\Longrightarrow \left.\frac{dR}{dE_R}\right|_{v^+_{min}} \leq \frac{dR_{tot}}{dE_R} \leq \left.\frac{dR}{dE_R}\right|_{v^-_{min}}$



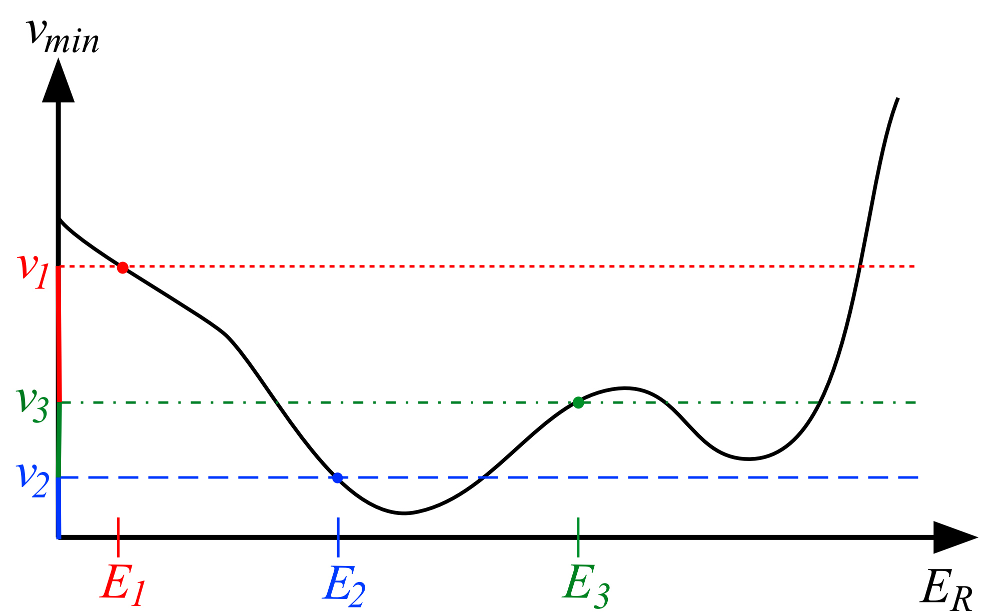


Model-independent bounds require many-to-1 v_{min}

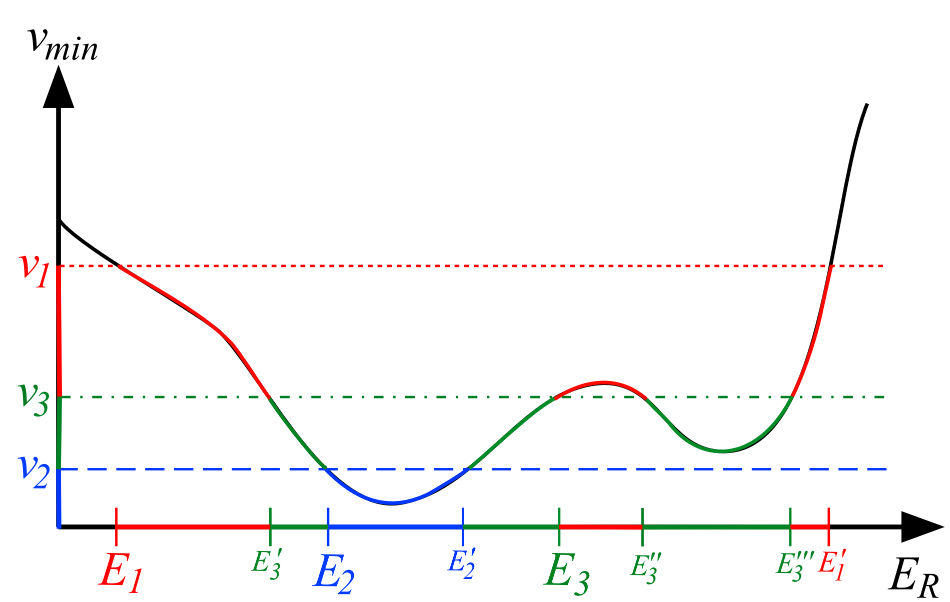
Effects of many-to-1 v_{min}

For a given form of $v_{min}(E_R)$,

mapping of measured E_R to v_{min} space is unambiguous…

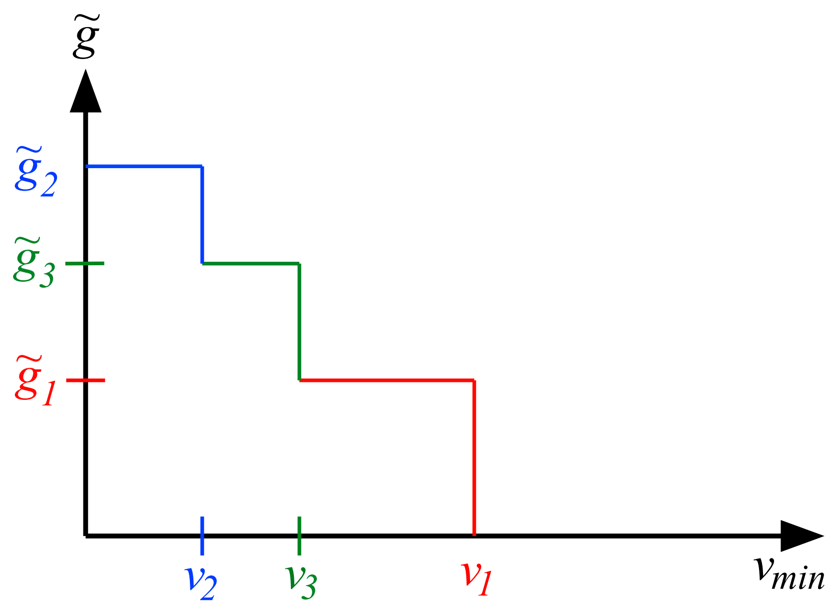


But mapping back to compute limits/best-fits is not

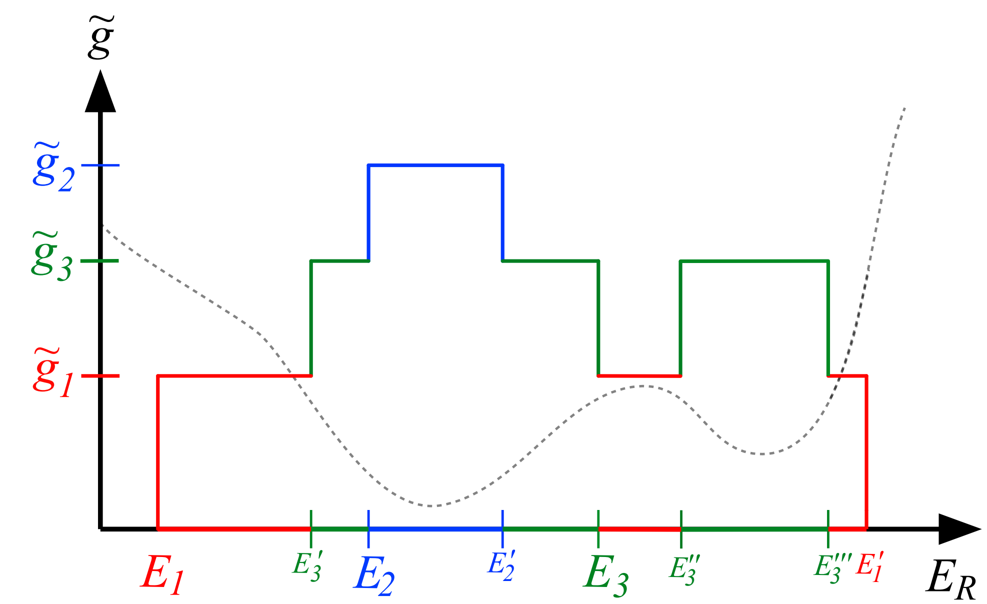


Halo-independent method for general kinematics

[YK, Fox, McCullough, 1409.xxxx]



unambiguous



(only a single step for setting limits)

Can prove as before that for sufficiently sharp energy resolution, $\tilde{g}(v_{min})$ still a sum of step functions

$$\left.\frac{dR}{dE_R}\right|_{E_i} = \frac{N_A m_n}{2\mu_{n\chi}^2} C_T^2(A,Z) \int dE_R' G(E_i, E_R') \epsilon(E_R') F^2(E_R') \tilde{g}(v_{min}(E_R'))$$

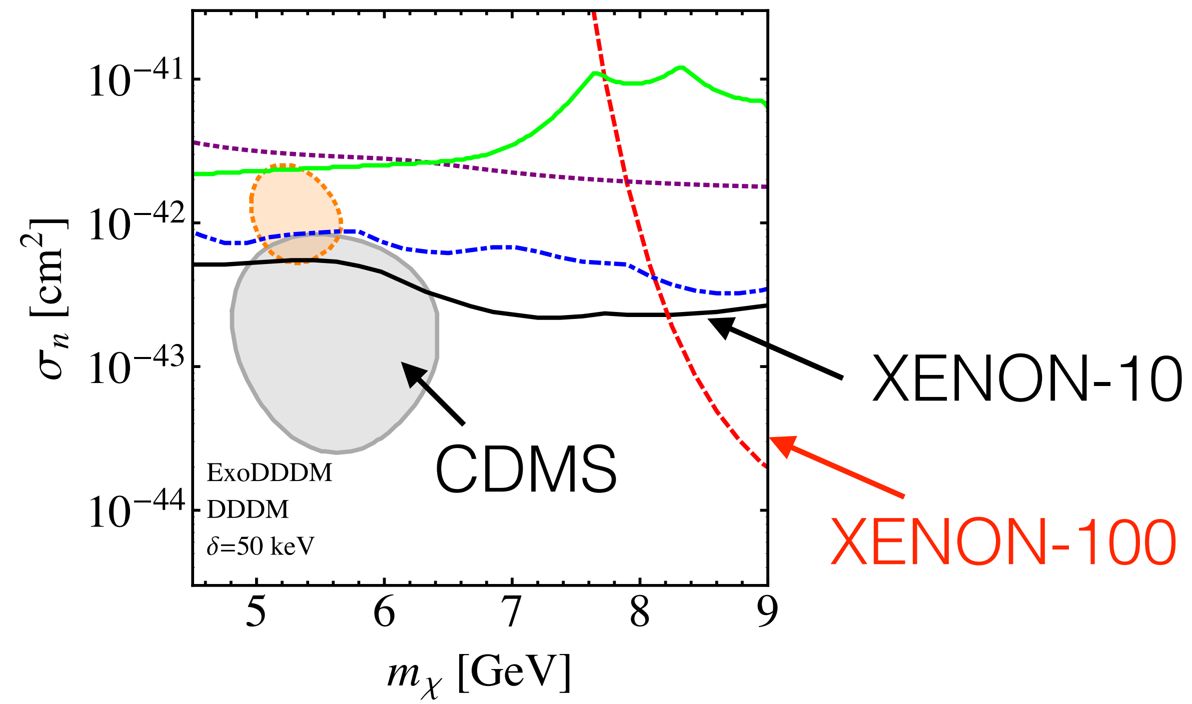
Plug in "unfolded" form for $\tilde{g}$, apply method exactly as before

Sample applications

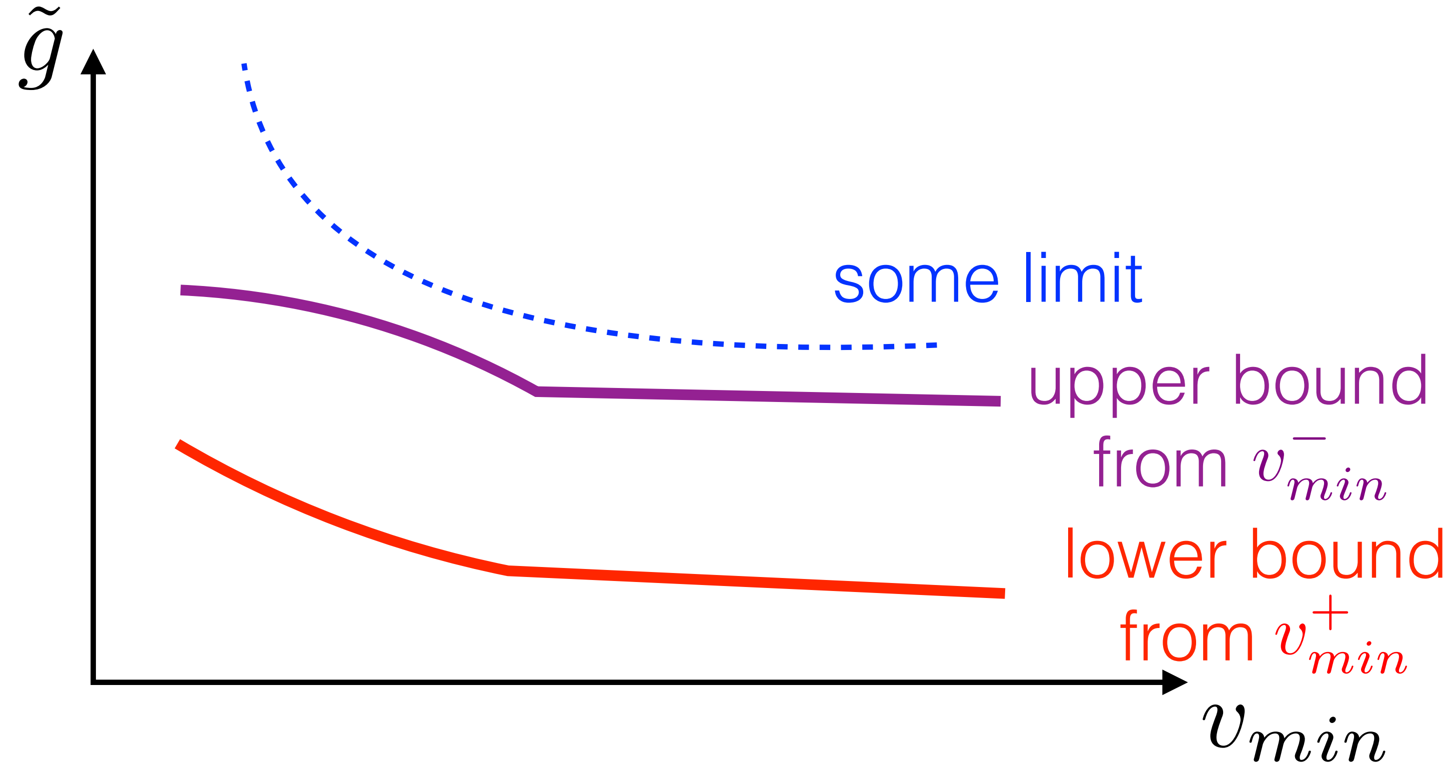
Exothermic: CDMS-Si vs. LUX and XENON

ExoDDDM w/standard halo
shown to be a good fit,
see if this holds with
halo-independent analysis

[McCullough and Randall, 1307.4095]

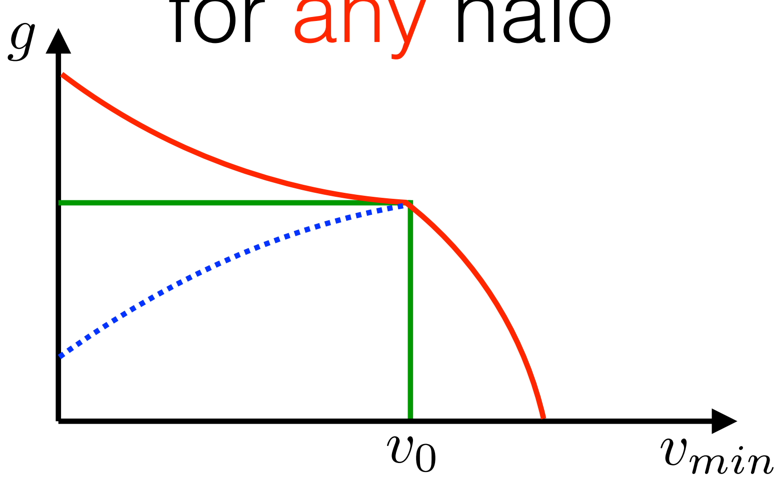


Multi-channel: model-independent limits and envelopes

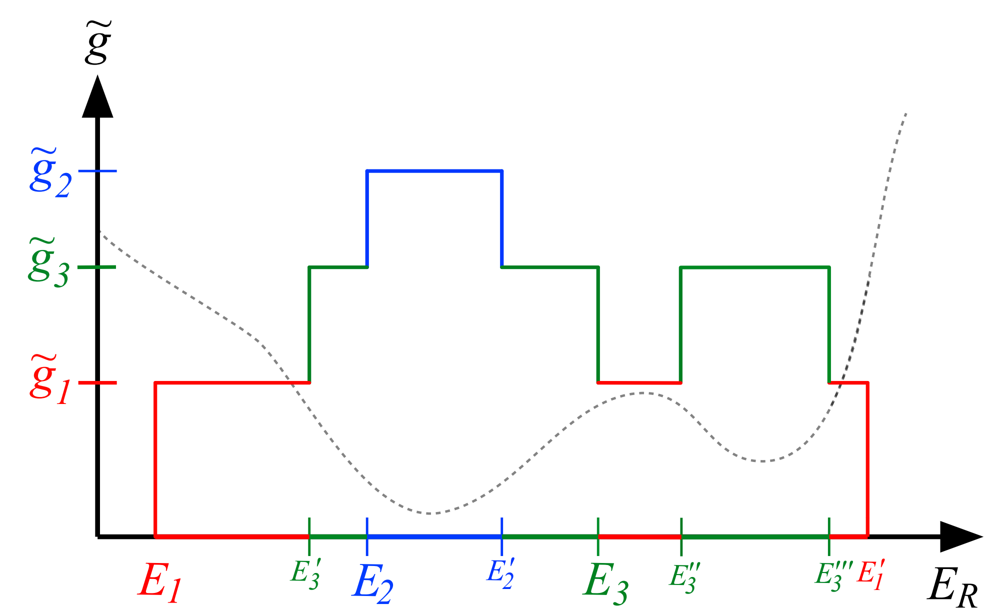


Summary

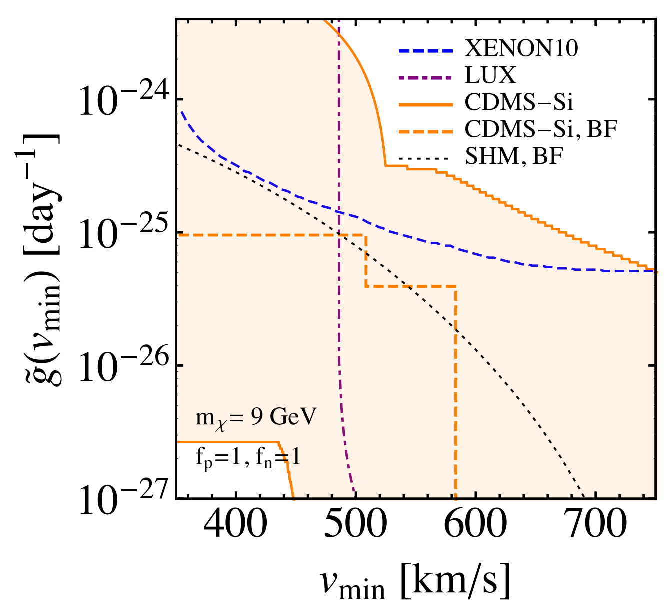
Monotonic velocity integral gives consistency conditions for any halo



Non-vanilla DM can give non-monotonic v_{min}, method still works!



Can apply halo-independent methods without binning



Conclusions and outlook

- DM direct detection making fantastic progress
- Emerging signals most likely to be seen in a small number of events, so need unbinned methods to maximize available information
- Still many DM unknowns, so extract information independent of DM halo and as agnostic as possible w.r.t. DM model (not necessarily elastic scattering!)
- Halo-independent methods are useful for experimentalists and theorists alike

Backup slides

Variational proof for monotonic v_{min}

$$L[\tilde{g}] = \int dE'_R\, K(E'_R)\tilde{g}(E'_R) - \sum_{i=1}^{N_O} \log\left(\mu_i + \int dE'_R\, G(E_i, E'_R)K(E'_R)\tilde{g}(E'_R)\right)$$

form factor, eff., etc.   BG rate

Monotonicity constraint: $d\tilde{g}/dE'_R \geq 0$

KKT conditions (Lagrange multipliers for inequality):

$$\frac{\delta L}{\delta \tilde{g}} - \frac{dq}{dE'_R} = 0\ ,$$ "E.O.M" for g

$$\frac{d\tilde{g}}{dE'_R} \leq 0\ ,$$ constraint

$$q(E'_R) \geq 0\ ,$$ positivity

$$\int dE'_R\, \frac{d\tilde{g}}{dE'_R} q(E'_R) = 0\ .$$ complementarity

Variational proof for monotonic v_{min}

$$L[\tilde{g}] = \int dE'_R \, K(E'_R)\tilde{g}(E'_R) - \sum_{i=1}^{N_O} \log\left(\mu_i + \int dE'_R \, G(E_i, E'_R)K(E'_R)\tilde{g}(E'_R)\right)$$

form factor, eff., etc.

BG rate

Assume nonzero derivative. E.O.M reduces to

$$\sum_{i=1}^{N_O} \frac{G(E_i, E_0)}{\gamma_i} = 1$$

some constant

But G is a resolution function and sharply peaked, so this only has discrete solutions - steps