Inflation and classical scale invariance

Antonio Racioppi

NICPB, Tallinn, Estonia

Euroopa Liit
Euroopa Sotsiaalfond

Eesti tuleviku heaks

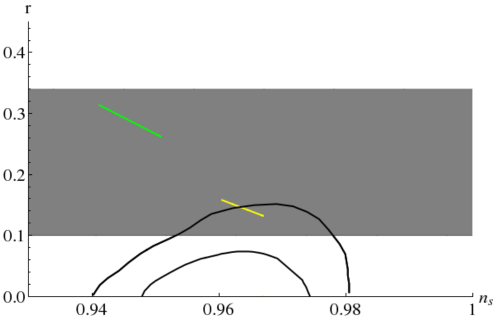
London, 19th August, 2014

based on JHEP 06 (2014) 154, 1408.xxxx
in collaboration with

K. Kannike (NICPB)
M. Raidal (NICPB & Institute of Physics, Tartu, Estonia)

Summary

Experimental Data



$\mathbf{r} = \frac{P_T(k)}{P_S(k)}$ — Planck bound — $V = m^2\phi^2$

$P_S(k) = k^{\mathbf{n_s}-1}$ BICEP2 2σ region — $V = \lambda\phi^4$

Classical scale invariance

From last year classical scale invariance is one of the hot topics in hep-ph...

1. A. G. Dias, A. F. Ferrari, J. D. Gomez, A. A. Natale and A. G. Quinto, arXiv:1407.1879 [hep-ph].
2. O. Antipin, E. Molgaard and F. Sannino, arXiv:1406.6166 [hep-th].
3. G. M. Pelaggi, arXiv:1406.4104 [hep-ph].
4. D. F. Litim and F. Sannino, arXiv:1406.2337 [hep-th].
5. V. V. Khoze and G. Ro, arXiv:1406.2291 [hep-ph].
6. S. Di Chiara, R. Foadi and K. Tuominen, arXiv:1405.7154 [hep-ph].
7. K. Kannike, A. Racioppi and M. Raidal, JHEP **1406** (2014) 154 [arXiv:1405.3987 [hep-ph]].
8. J. Kubo, K. S. Lim and M. Lindner, arXiv:1405.1052 [hep-ph].
9. A. Farzinnia and J. Ren, Phys. Rev. D **90** (2014) 015019 [arXiv:1405.0498 [hep-ph]].
10. H. Davoudiasl and I. M. Lewis, arXiv:1404.6260 [hep-ph].
11. A. Kobakhidze and K. L. McDonald, arXiv:1404.5823 [hep-ph].
12. J. Kubo, K. S. Lim and M. Lindner, arXiv:1403.4262 [hep-ph].
13. A. Salvio and A. Strumia, JHEP **1406** (2014) 080 [arXiv:1403.4226 [hep-ph]].
14. A. Fowlie, arXiv:1403.3407 [hep-ph].
15. A. de Gouvea, D. Hernandez and T. M. P. Tait, Phys. Rev. D **89** (2014) 115005 [arXiv:1402.2658 [hep-ph]].
16. S. Benic and B. Radovcic, Phys. Lett. B **732** (2014) 91 [arXiv:1401.8183 [hep-ph]].
17. M. Hashimoto, S. Iso and Y. Orikasa, Phys. Rev. D **89** (2014) 056010 [arXiv:1401.5944 [hep-ph]].
18. J. Guo and Z. Kang, arXiv:1401.5609 [hep-ph].
19. S. Abel and A. Mariotti, arXiv:1312.5335 [hep-ph].
20. J. M. Cline, Z. Liu, G. Moore and W. Xue, arXiv:1312.3325 [hep-ph].
21. Y. Kawamura, arXiv:1311.2365 [hep-ph].
22. J. D. Clarke, R. Foot and R. R. Volkas, JHEP **1402** (2014) 123 [arXiv:1310.8042 [hep-ph]].
23. M. Lindner, D. Schmidt and A. Watanabe, Phys. Rev. D **89** (2014) 1, 013007 [arXiv:1310.6582 [hep-ph]].
24. M. Holthausen, J. Kubo, K. S. Lim and M. Lindner, JHEP **1312** (2013) 076 [arXiv:1310.4423 [hep-ph]].
25. M. Hashimoto, S. Iso and Y. Orikasa, Phys. Rev. D **89** (2014) 016019 [arXiv:1310.4304 [hep-ph]].

and many more . . . sorry for any missing references ⌣̈

Classical scale invariance

Following the standard Wilsonian prescription,

$$V = V_{\rm ren} + \sum_{n=5}^{\infty} c_n \frac{\phi^n}{M_P^{n-4}}$$

- $V_{\rm ren}$ is the renormalisable part of inflaton potential
- $M_P \simeq 2.4 \times 10^{18}$ GeV is the reduced Planck mass
- c_n are the Wilson coefficients of gravity-induced higher order operators.

Lyth bound (Phys. Rev. Lett. 78, 1861) $\Rightarrow \Delta\phi/M_P > 1 \Rightarrow V \gg (10^{16}\ \mathrm{GeV})^4$

$$\Downarrow$$

$$V^* \simeq \left(1.94 \times 10^{16}\mathrm{GeV}\right)^4 r^*/0.12 \not\Leftrightarrow \text{Planck/BICEP2 data}$$

apparent absence of the Planck scale induced operators $\leftrightarrow$ classically scale-free fundamental physics $\Rightarrow$ all mass scales generated by quantum effects

Dimensional transmutation

Tree level: $V_\phi = \frac{1}{4}\lambda_\phi \phi^4 \quad \Rightarrow \quad V_\phi^{\min} : \langle\phi\rangle = v_\phi = 0$

RGE : $\beta_{\lambda_\phi} = \frac{d\lambda_\phi}{d\ln\mu}$

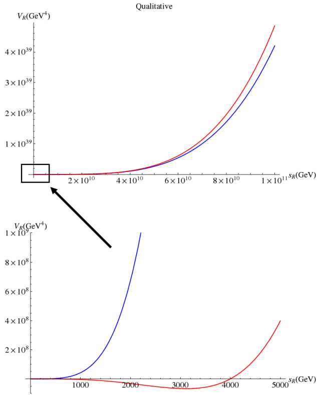
$\Rightarrow$ if: $\beta_{\lambda_\phi} \sim$ const. (at least in some region) and > 0

$$\Rightarrow \int_{\lambda_{\phi_0}}^{\lambda_\phi} d\lambda_\phi \simeq \beta_{\lambda_\phi} \int_{\ln\mu_0}^{\ln\mu} d(\ln\mu)$$

$$\Rightarrow \quad \lambda_\phi \simeq \lambda_{\phi_0} + \beta_{\lambda_\phi} \ln\frac{\mu}{\mu_0}$$

$\Rightarrow$ fixing $\lambda_{\phi_0} = 0$

$$\Rightarrow \boxed{\lambda_\phi \simeq \beta_{\lambda_\phi} \ln\frac{\mu}{\mu_0}} \rightarrow \begin{cases} \lambda_\phi(\mu > \mu_0) > 0 \\ \lambda_\phi(\mu < \mu_0) < 0 \end{cases}$$



One loop: $V_{R,1\text{loop}} \simeq \frac{1}{4}\beta_{\lambda_\phi} \ln\frac{|\phi|}{\phi_0}\phi^4 \quad \Rightarrow \quad v_\phi \simeq \phi_0 e^{-1/4} \qquad (\phi_0 \leftrightarrow \mu_0)$

Coleman-Weinberg inflation

- Already the first papers on inflation considered CW inflation:
  Linde, Phys. Lett. B108 (1982) 389-393, Phys. Lett. B114 (1982) 431; Albrecht and Steinhardt, Phys. Rev. Lett. 48 (1982) 1220-1223; Ellis et al., Nucl. Phys. B221 (1983) 524, Phys. Lett. B120 (1983) 331.
- This idea has been (is being) extensively studied in the context of
  - GUT:
    Langbeine et al, Mod.Phys.Lett. A11 (1996) 631-646; Gonzalez-Diaz, Phys.Lett. B176 (1986) 29-32; Yokoyama, Phys.Rev. D59 (1999) 107303; Rehman et al., Phys.Rev. D78 (2008) 123516.
  - $U(1)_{B-L}$:
    Barenboim et al., Phys.Lett. B730 (2014) 81-88; N. Okada and Q. Shafi, 1311.0921.
  - $SU(N)$:
    Elizalde et al., 1408.1285.
- They all suppose new gauge groups beyond the SM: NO NEED FOR IT!
- It can occur just due to running of some scalar quartic coupling, to negative values at some energy scale due to couplings to other scalar fields, generating non-trivial physical potentials.

Models proposed

A. QFT classical scale invariant, but not GR: M_P, Λ by hand.

- Inflaton minimally coupled to gravity: $\xi = 0$,
  JHEP 06 (2014) 154

B. QFT and GR classical scale invariant: M_P induced by v_ϕ

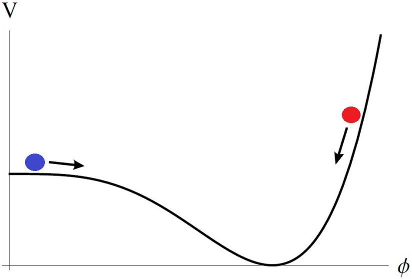
- Inflaton non-minimally coupled to gravity: $\xi \neq 0$
  1408.xxxx

Model A: M_P by hand. Framework

$$V_{\text{eff}}(\phi) = \frac{\lambda_\phi}{4}\phi^4 + \Lambda^4$$

$$\lambda_\phi = \beta_{\lambda_\phi} \ln\left|\frac{\phi}{\phi_0}\right|$$

$$V_{\text{eff}}(v_\phi) = 0 \Rightarrow \Lambda = \phi_0 \sqrt[4]{\frac{\beta_{\lambda_\phi}}{16e}}$$



Such a shape allows for two different, generic types of inflation:

i. Small-field (hilltop) inflation, when ϕ rolls forward down to v_ϕ: ●

ii. Large-field (chaotic) inflation, when ϕ rolls back down to v_ϕ: ●

Recipe for studying inflation

$$S = \int d^4x\sqrt{-g}\left(\frac{M_P^2}{2}R + \mathcal{L}_\phi\right) \qquad \mathcal{L}_\phi = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi)$$

1. Calculate the field value at the end of inflation ϕ_e from

$$\epsilon\left(\phi_e\right) = \frac{M_{\mathrm{P}}^2}{2}\left(\frac{V'\left(\phi_e\right)}{V\left(\phi_e\right)}\right)^2 = 1$$

2. Calculate ϕ^*, N e-folds before the end of inflation,

$$N = \frac{1}{M_{\mathrm{P}}}\int_{\phi_e}^{\phi^*}\frac{d\phi'}{\sqrt{2\epsilon\left(\phi'\right)}}$$

Recipe for studying inflation

3. Calculate r and n_S at $N = [50, 60]$ by

$$\begin{aligned} r &= 16\epsilon\left(\phi^*\right) \\ n_s &= 1 - 6\epsilon\left(\phi^*\right) + 2\eta\left(\phi^*\right) \end{aligned}$$

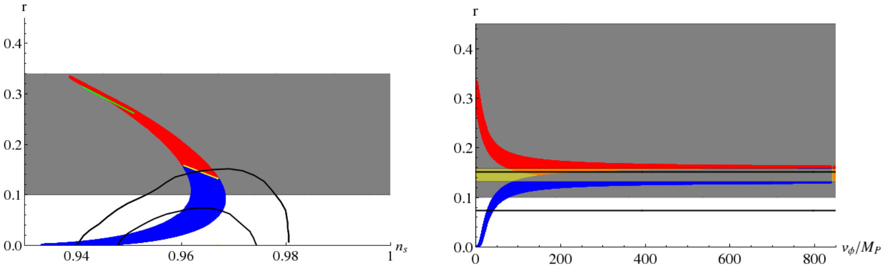
where the second slow roll parameter η is given by:

$$\eta\left(\phi^*\right) = M_{\mathrm{P}}^2 \frac{V''\left(\phi^*\right)}{V\left(\phi^*\right)}$$

4. From the scalar amplitude measurement: $A_s^2 = \frac{V(\phi)}{24\pi^2 M_P^4 \epsilon(\phi^*)} \approx 2.45 \times 10^{-9}$ fix the overall normalization of V:

$$V(\phi^*) \simeq \left(1.94 \times 10^{16}\ \text{GeV}\right)^4 \frac{r}{0.12}$$

Model A: M_P by hand. Results



– Planck bound | — $V = m^2\phi^2$ | ■ chaotic

■ BICEP2 2σ region | — $V = \lambda\phi^4$ | ■ hilltop

- hilltop case favoured
- $v_\phi \gg M_P \sim V = m^2\phi^2$: V symmetric around the v_ϕ and loss of sensitivity to the initial conditions.
- $r \ll 0.1$ and $n_s < 0.945$. $\leftrightarrow$ Barenboim et al., Phys.Lett. B730 (2014) $\Rightarrow$ ruled out by Planck data because of "restrictive" assumption: $v_\phi < M_P \rightarrow$ solved by S. Iso et al., 1408.2339

Model A: M_P by hand. Realization

$$\mathcal{L}_{\text{matter}} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi + \frac{1}{2}\partial_\mu\eta\partial^\mu\eta + \mathcal{L}_Y - V$$

$$\mathcal{L}_Y = Y_\phi^{ij}\bar{N}_i^c N_j\phi + Y_\eta^{ij}\bar{N}_i^c N_j\eta$$

$$V = \frac{\lambda_\phi}{4}\phi^4 + \frac{\lambda_{\phi\eta}}{4}\eta^2\phi^2 + \frac{\lambda_\eta}{4}\eta^4 + \Lambda^4$$

- ϕ (inflaton) and η: two real singlet scalar field
- N_i: three heavy singlet right-handed neutrinos
- no explicit mass terms in the scalar potential V

Model A: M_P by hand. Realization

The one-loop inflaton effective potential: $V_{\rm eff} = V + \Delta V$

$$\Delta V = \frac{1}{64\pi^2}\left[\sum_{i=1}^{2} m_i^4\left(\ln\frac{m_i^2}{\mu^2} - \frac{3}{2}\right) - 2\mathrm{Tr}\left\{M_N M_N^\dagger\left(\ln\frac{M_N M_N^\dagger}{\mu^2} - \frac{3}{2}\right)\right\}\right]$$

- m_i^2: eigenvalues of $m^2_{\phi\eta} = \begin{pmatrix} 3\lambda_\phi\phi^2 + \frac{1}{2}\lambda_{\phi\eta}\eta^2 & \lambda_{\phi\eta}\phi\eta \\ \lambda_{\phi\eta}\phi\eta & 3\lambda_\eta\eta^2 + \frac{1}{2}\lambda_{\phi\eta}\phi^2 \end{pmatrix}$
- $M_N = Y_\phi\phi + Y_\eta\eta$
- μ is the renormalisation scale

Model A: M_P by hand. Realization

- Inflation will take place in the direction $\eta = 0$, which is the minimal value for the field η. We will see later that such an assumption is self-consistent.

- We neglect:
  a. higher order λ_ϕ contribution (since it must be extremely small)
  b. the heavy neutrino contributions at one loop level.

- The RGE improved effective potential for the direction of ϕ reads

$$V_{\rm eff} = \frac{\lambda_\phi(\mu)\phi^4}{4} + \frac{\lambda_{\phi\eta}^2}{256\pi^2}\left(\ln\frac{\phi^2\lambda_{\phi\eta}}{2\mu^2} - \frac{3}{2}\right)\phi^4 + \Lambda^4$$

Model A: M_P by hand. Realization

- The beta function β_{λ_ϕ}:

$$16\pi^2\beta_{\lambda_\phi} = 18\lambda_\phi^2 + \frac{1}{2}\lambda_{\phi\eta}^2 + 16\lambda_\phi \mathrm{Tr}\, Y_\phi^\dagger Y_\phi - 64 \mathrm{Tr}\, Y_\phi Y_\phi^\dagger Y_\phi Y_\phi^\dagger$$

$$\Rightarrow \text{ neglect: } \lambda_\phi, \ \ Y_\phi \Rightarrow \boxed{\beta_{\lambda_\phi} \simeq \frac{\lambda_{\phi\eta}^2}{32\pi^2}}$$

- $\lambda_\phi(\mu) = \frac{\lambda_{\phi\eta}^2}{64\pi^2}\ln\frac{\mu^2}{\mu_0^2} \quad \rightarrow \quad V_{\text{eff}} = \frac{\lambda_{\phi\eta}^2}{256\pi^2}\left(\ln\frac{\mu^2}{\mu_0^2} + \ln\frac{\lambda_{\phi\eta}\phi^2}{2\mu^2} - \frac{3}{2}\right)\phi^4 + \Lambda^4$

- We can eliminate the μ dependence

$$V_{\text{eff}} = \frac{\lambda_{\phi\eta}^2}{256\pi^2}\left[\ln\left(\frac{\lambda_{\phi\eta}\phi^2}{2\mu_0^2}\right) - \frac{3}{2}\right]\phi^4 + \Lambda^4$$

Model A: M_P by hand. Realization

- After a simple reparametrization

$$V_{\rm eff} = \frac{\lambda_{\phi\eta}^2 \ln \frac{\phi^2}{\phi_0^2}}{256\pi^2}\phi^4 + \Lambda^4$$

where $\phi_0 = \sqrt{\frac{2e^{3/2}}{\lambda_{\phi\eta}}}\mu_0$

- which is equivalent of

$$\begin{aligned} V_{\rm eff} &= \frac{\lambda_\phi(\mu)}{4}\phi^4 + \Lambda^4 \\ \lambda_\phi(\mu) &= \beta_{\lambda_\phi} \ln\left|\frac{\mu}{\mu_0}\right| \\ \beta_{\lambda_\phi} &= \frac{\lambda_{\phi\eta}^2}{32\pi^2} \\ \mu,\ \mu_0 &\rightarrow \phi,\ \phi_0 \end{aligned}$$

Model A: M_P by hand. Results II

- $m_\phi \sim 10^{13}$ GeV
- In the allowed region: $\lambda(\phi)^* < 0$.
  - hilltop: $0 > \lambda(\phi)^* > -2 \times 10^{-14}$
  - chaotic: $0 > \lambda(\phi)^* > -3 \times 10^{-16}$
- $\lambda_{\phi\eta} \lesssim 10^{-5}$: small enough to be treated as a constant $\Rightarrow \beta_{\lambda_\phi} \propto \lambda_{\phi\eta}^2$ constant.
- For consistency, $\lambda_\eta > 0$, but otherwise can have any value.
- $m_\eta = \sqrt{\frac{\lambda_{\phi\eta}}{2}} v_\phi \sim 10^{17}$ GeV
  - $m_\eta > V^* \sim 10^{16}$ GeV, and $m_\eta \gg m_\phi$ $\Rightarrow \eta$ is decoupled and is frozen at $\eta = 0$ during inflation.
  - Lebedev and Westphal, Phys.Lett. B719 (2013) 415-418: $\eta = 0$ is quickly achieved during inflation due to $\lambda_{\phi\eta} \Rightarrow$ model is self-consistent.
- $\phi \to NN \Leftrightarrow m_\phi > 2m_N \Rightarrow Y_\phi$ neglectable in RGE, $T_{RH} \sim 10^7$ GeV

Model B: Induced M_P and ξ. Framework

$$S = \int d^4x\sqrt{-g}\left[f(\phi)R + \mathcal{L}_{\text{matter}}\right] \qquad f(\phi) = \frac{\xi_\phi}{2}\phi^2$$

$$\mathcal{L}_{\text{matter}} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi + \frac{1}{2}\partial_\mu\eta\partial^\mu\eta + \mathcal{L}_Y - V$$

$$\mathcal{L}_Y = Y_\phi^{ij}\bar{N}_i^c N_j\phi + Y_\eta^{ij}\bar{N}_i^c N_j\eta$$

$$V = \frac{\lambda_\phi}{4}\phi^4 + \frac{\lambda_{\phi\eta}}{4}\eta^2\phi^2 + \frac{\lambda_\eta}{4}\eta^4$$

- Full classical scale invariance $\rightarrow M_P$ dynamically
- The running of λ_ϕ allows $v_\phi \neq 0 \Rightarrow$

$$\Rightarrow \quad f(\phi) \rightarrow f(\varphi + v_\phi) = \xi_\phi\left(\varphi + v_\phi\right)^2/2 \quad \Rightarrow \quad \boxed{M_P^2 = \xi_\phi v_\phi^2}$$

- $\eta = 0$ during inflation

From the Jordan frame to the Einstein frame

- Conformal transformation:

$$g_{\mu\nu} \to \Omega(\phi)^2 g_{\mu\nu}$$
$$\Omega(\phi)^2 = \frac{2}{M_P^2} f(\phi)$$

- The scalar potential in the Einstein frame is given by

$$U = \frac{V(\phi)}{\Omega(\phi)^4} = \frac{\lambda_\phi(\phi)\phi^4}{4\Omega(\phi)^4}$$

- Canonically normalised field χ

$$\frac{d\chi}{d\phi} = M_P \sqrt{\frac{f(\phi) + 3f(\phi)'^2}{2f(\phi)^2}}$$

Model B: Induced M_P and ξ. Scalar potential $U(\phi)$

- Since we live in the minimum with non-zero Planck scale: $\phi = \varphi + v_\phi$.

$$\left.\begin{aligned} V(\phi) &= \lambda_\phi(\varphi + v_\phi)\,(\varphi + v_\phi)^4 \\ f(\phi) &= \tfrac{1}{2}(\varphi + v_\phi)^2 \end{aligned}\right\} \Rightarrow \boxed{U = \frac{1}{4}\lambda_\phi(\varphi + v_\phi)\frac{M_P^4}{\xi_\phi^2}}$$

- Slow-roll parameters

$$\epsilon = \frac{M_P^2}{2}\left(\frac{U'}{U}\frac{1}{d\chi/d\phi}\right)^2$$

$$\eta = \frac{M_P^2}{U}\frac{d^2U}{d\chi^2} = \frac{M_P^2}{U}\left[U''\left(\frac{d\chi}{d\phi}\right) - U'\left(\frac{d\chi}{d\phi}\right)'\right] \Big/ \left(\frac{d\chi}{d\phi}\right)^3$$

- $N = \dfrac{1}{\sqrt{2}M_P}\displaystyle\int_{\phi_{\rm end}}^{\phi^*}\frac{d\phi}{\sqrt{\epsilon}}$ where $d\phi = \dfrac{1}{d\chi/d\phi}$

- now we can apply the recipe given before

Model B: Induced M_P and ξ. Scalar potential $U(\phi)$

- full classical scale invariance $\rightarrow U(v_\phi) \sim 0$
- induce the minimum in U via a minimum in $\lambda_\phi(\phi)$.
- Around the VEV, we can Taylor expand:

$$\lambda_\phi(\phi) = \lambda_\phi(v_\phi) + \lambda'_\phi(v_\phi)(\phi - v_\phi) + \frac{1}{2}\lambda''_\phi(v_\phi)(\phi - v_\phi)^2 + \mathcal{O}(\phi^3)$$

  i. $\lambda_\phi(v_\phi) = 0 \rightarrow$ ensure a small vacuum energy after inflation (condition on the RGE of λ_ϕ)
  ii. $\lambda'_\phi(v_\phi) = 0 \rightarrow$ minimum of $\lambda \Rightarrow$ minimum of U (condition on the RGE of $\lambda_{\phi\eta}$ and Y_ϕ: N contribution crucial!)

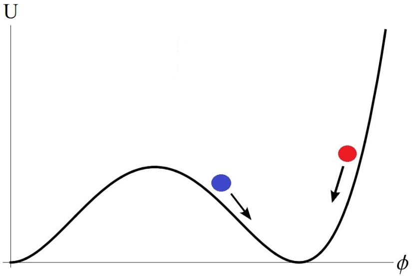
- Old concept: Multiple Point Criticality Principle. Used for predicting the Higgs mass: C. D. Froggatt, H. B. Nielsen, Phys.Lett. B368 (1996)
- Same idea already applied for Higgs inflation: Haba et al. 1406.0158

Model B: Induced M_P and ξ. Shape of potential

$$S = \int d^4x\sqrt{-g}\left[f(\phi)R + \mathcal{L}_{\rm matter}\right]$$

$$f(\phi) = \frac{\xi_\phi}{2}\phi^2$$

$\mathcal{L}_{\rm matter} =$ same as before



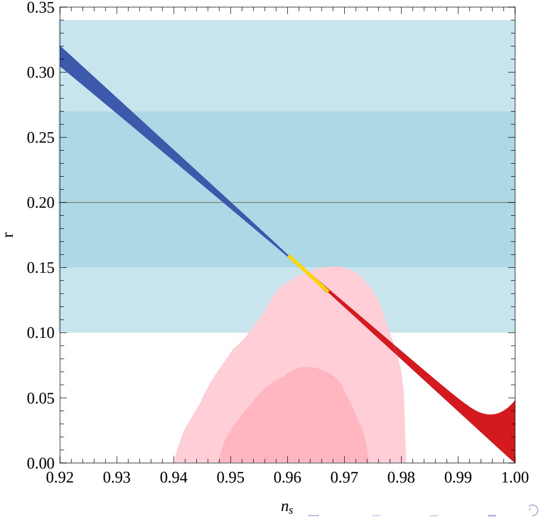
- Such a shape allows for two different, generic types of inflation:
    i. Small-field inflation, when ϕ rolls forward down to v_ϕ: ●
    ii. Large-field (chaotic) inflation, when ϕ rolls back down to v_ϕ: ●

Model B: Induced M_P and ξ. Results

$N \in [50, 60]$

$\xi_\phi \in [0, 0.0115]$

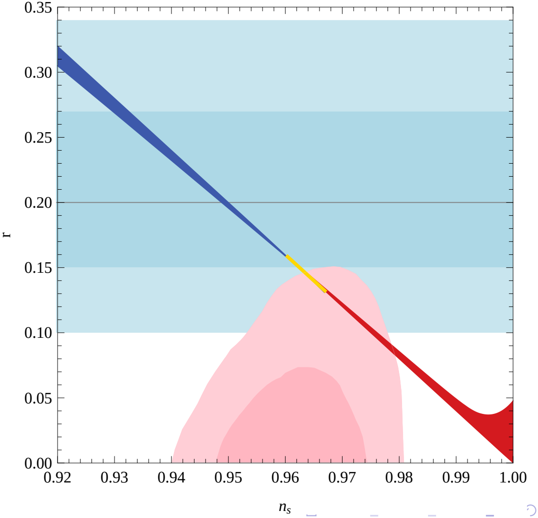
- Planck bound
- BICEP2 bound
- $V = m^2\phi^2$
- large field
- small field



Model B: Induced M_P and ξ. Other results

In the allowed region:

- large field case favoured
- $m_\phi \sim 10^{13}$ GeV
- $\lambda_{\phi\eta} \sim 10^{-4}$
- $\xi_\phi \sim 10^{-3}$
- $m_\eta \sim M_P$ GeV
- $T_{RH} \sim 10^{12,13}$ GeV



Conclusions

- Classical scale invariance applied to inflation can bring interesting and different results according to how you implement it.
- Model A: M_P by hand
  a. Found region in agreement with BICEP2 & Planck
  b. small field case favoured
  c. V for $v_\phi \to \infty \sim m^2\phi^2$
- Model B: Induced M_P and ξ
  a. Found region in agreement with BICEP2 & Planck
  b. large field case favoured
  c. V for $v_\phi \to \infty \sim m^2\phi^2$
- Wait for updated data

Thank you!

Backup slides

Full Lagrangian

$$\mathcal{L} = \mathcal{L}_{SM} + \frac{1}{2}\partial_\mu\phi\partial^\mu\phi + \frac{1}{2}\partial_\mu\eta\partial^\mu\eta + \mathcal{L}_Y - V$$

$$\mathcal{L}_Y = Y_N^{ij}\bar{L}_i i\sigma_2 H^* N_j + \mathrm{h.c.} + Y_\phi^{ij}\bar{N}_i^c N_j\phi + Y_\eta^{ij}\bar{N}_i^c N_j\eta$$

$$V = \Lambda^4 + \frac{1}{2}\lambda_{h\phi}|H|^2\phi^2 + \frac{1}{2}\lambda_{h\eta}|H|^2\eta^2 + \frac{\lambda_\phi}{4}\phi^4 + \frac{\lambda_{\phi\eta}}{4}\eta^2\phi^2 + \frac{\lambda_\eta}{4}\eta^4,$$

RGEs I

We derived the RGEs using the PyR@TE package:
F. Lyonnet et al., Comput.Phys.Commun. 185 (2014) 1130-1152.

$$16\pi^2\beta_{\lambda_h} = \beta^{\rm SM}_{\lambda_h} + \frac{1}{2}(\lambda_{h\phi}^2 + \lambda_{h\eta}^2) + 2\lambda_h(\mathrm{Tr}Y_N^\dagger Y_N + \mathrm{Tr}Y_N^* Y_N^T) - \mathrm{Tr}Y_N Y_N^\dagger Y_N Y_N^\dagger - \mathrm{Tr}Y_N^T Y_N^* Y_N^T Y_N^*$$

$$16\pi^2\beta_{\lambda_{h\phi}} = 4\lambda_{h\phi}^2 + \lambda_{\phi\eta}\lambda_{h\eta} + 6\lambda_{h\phi}(2\lambda_h + \lambda_\phi) - \lambda_{h\phi}\left(\frac{3}{2}g^2 + \frac{9}{2}g'^2\right) + \lambda_{h\phi}[\mathrm{Tr}Y_e^\dagger Y_e + \mathrm{Tr}Y_e^* Y_e^T + 3\mathrm{Tr}Y_d^\dagger Y_d + 3\mathrm{Tr}Y_d^* Y_d^T + 3\mathrm{Tr}Y_u^\dagger Y_u + 3\mathrm{Tr}Y_u^* Y_u^T + \mathrm{Tr}Y_N^* Y_N^T + \mathrm{Tr}Y_N^\dagger Y_N + 8\mathrm{Tr}Y_\phi^\dagger Y_\phi] - 8\mathrm{Tr}Y_\phi Y_N^\dagger Y_N Y_\phi^\dagger - 8\mathrm{Tr}Y_\phi Y_\phi^\dagger Y_N^T Y_N^* - 8\mathrm{Tr}Y_N Y_\phi^\dagger Y_\phi Y_N^\dagger - 8\mathrm{Tr}Y_N^T Y_N^* Y_\phi Y_\phi^\dagger,$$

$$16\pi^2\beta_{\lambda_{h\eta}} = 4\lambda_{h\eta}^2 + \lambda_{\eta\eta}\lambda_{h\eta} + 6\lambda_{h\eta}(2\lambda_h + \lambda_\eta) - \lambda_{h\eta}\left(\frac{3}{2}g^2 + \frac{9}{2}g'^2\right) + \lambda_{h\eta}[\mathrm{Tr}Y_e^\dagger Y_e + \mathrm{Tr}Y_e^* Y_e^T + 3\mathrm{Tr}Y_d^\dagger Y_d + 3\mathrm{Tr}Y_d^* Y_d^T + 3\mathrm{Tr}Y_u^\dagger Y_u + 3\mathrm{Tr}Y_u^* Y_u^T + \mathrm{Tr}Y_N^* Y_N^T + \mathrm{Tr}Y_N^\dagger Y_N + 8\mathrm{Tr}Y_\eta^\dagger Y_\eta] - 8\mathrm{Tr}Y_\eta Y_N^\dagger Y_N Y_\eta^\dagger - 8\mathrm{Tr}Y_\eta Y_\eta^\dagger Y_N^T Y_N^* - 8\mathrm{Tr}Y_N Y_\eta^\dagger Y_\eta Y_N^\dagger - 8\mathrm{Tr}Y_N^T Y_N^* Y_\eta Y_\eta^\dagger,$$

RGEs II

$$16\pi^2\beta_{\lambda_\phi} = 18\lambda_\phi^2 + \frac{1}{2}\lambda_{\phi\eta}^2 + 2\lambda_{h\phi}^2 + 16\lambda_\phi \mathrm{Tr} Y_\phi^\dagger Y_\phi - 64\mathrm{Tr} Y_\phi Y_\phi^\dagger Y_\phi Y_\phi^\dagger ,$$

$$16\pi^2\beta_{\lambda_\eta} = 18\lambda_\eta^2 + \frac{1}{2}\lambda_{\phi\eta}^2 + 2\lambda_{h\eta}^2 + 16\lambda_\eta \mathrm{Tr} Y_\eta^\dagger Y_\eta - 64\mathrm{Tr} Y_\eta Y_\eta^\dagger Y_\eta Y_\eta^\dagger ,$$

$$\begin{aligned}16\pi^2\beta_{\lambda_{\phi\eta}} &= 4\lambda_{\phi\eta}^2 + 4\lambda_{h\eta}\lambda_{h\phi} + 6\lambda_\phi\lambda_{\phi\eta} + 6\lambda_\eta\lambda_{\phi\eta} + 8\lambda_{\phi\eta}\mathrm{Tr} Y_\eta^\dagger Y_\eta + 8\lambda_{\phi\eta}\mathrm{Tr} Y_\phi^\dagger Y_\phi \\ &- 64\mathrm{Tr} Y_\eta Y_\eta^\dagger Y_\phi Y_\phi^\dagger - 64\mathrm{Tr} Y_\phi Y_\eta^\dagger Y_\eta Y_\phi^\dagger - 64\mathrm{Tr} Y_\eta Y_\phi^\dagger Y_\phi Y_\eta^\dagger - 64\mathrm{Tr} Y_\phi Y_\phi^\dagger Y_\eta Y_\eta^\dagger \\ &- 64\mathrm{Tr} Y_\phi Y_\eta^\dagger Y_\phi Y_\eta^\dagger - 64\mathrm{Tr} Y_\eta Y_\phi^\dagger Y_\eta Y_\phi^\dagger ,\end{aligned}$$

RGEs III

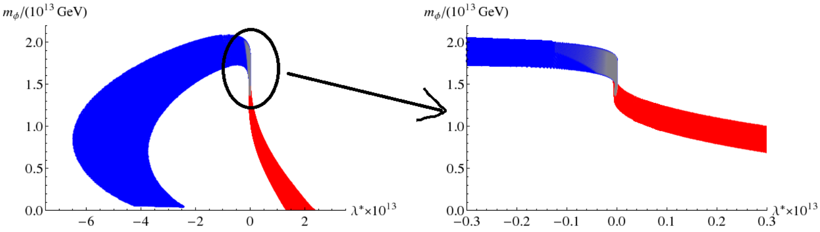
$$16\pi^2\beta_{Y_N} = -\frac{3}{4}g^2 Y_N - \frac{9}{4}g'^2 Y_N + \frac{3}{2}\mathrm{Tr}\left(Y_u^\dagger Y_u\right)Y_N + 2Y_N Y_\phi^\dagger Y_\phi + \frac{3}{2}Y_N Y_N^\dagger Y_N$$
$$- \frac{3}{2}Y_e Y_e^\dagger Y_N + \frac{1}{2}\mathrm{Tr}\left(Y_e^\dagger Y_e\right)Y_N + \frac{3}{2}\mathrm{Tr}\left(Y_d^\dagger Y_d\right)Y_N + 2Y_N Y_\eta^\dagger Y_\eta + \frac{3}{2}\mathrm{Tr}\left(Y_d^* Y_d^T\right)Y_N$$
$$+ \frac{1}{2}\mathrm{Tr}\left(Y_e^* Y_e^T\right)Y_N + \frac{3}{2}\mathrm{Tr}\left(Y_u^* Y_u^T\right)Y_N + \frac{1}{2}\mathrm{Tr}\left(Y_N^\dagger Y_N\right)Y_N + \frac{1}{2}\mathrm{Tr}\left(Y_N^* Y_N^T\right)Y_N,$$

$$16\pi^2\beta_{Y_\phi} = 4\mathrm{Tr}\left(Y_\phi^\dagger Y_\phi\right)Y_\phi + 8Y_\eta Y_\phi^\dagger Y_\eta + 2\mathrm{Tr}\left(Y_\eta^\dagger Y_\phi\right)Y_\eta + 2\mathrm{Tr}\left(Y_\phi^\dagger Y_\eta\right)Y_\eta$$
$$+ Y_N^T Y_N^* Y_\phi + 2Y_\eta Y_\eta^\dagger Y_\phi + Y_\phi Y_N^\dagger Y_N + 12Y_\phi Y_\phi^\dagger Y_\phi + Y_\phi Y_\eta^\dagger Y_\eta,$$

$$16\pi^2\beta_{Y_\eta} = 4\mathrm{Tr}\left(Y_\eta^\dagger Y_\eta\right)Y_\eta + 8Y_\phi Y_\eta^\dagger Y_\phi + 2\mathrm{Tr}\left(Y_\phi^\dagger Y_\eta\right)Y_\phi + 2\mathrm{Tr}\left(Y_\eta^\dagger Y_\phi\right)Y_\phi$$
$$+ Y_N^T Y_N^* Y_\eta + 2Y_\phi Y_\phi^\dagger Y_\eta + Y_\eta Y_N^\dagger Y_N + 12Y_\eta Y_\eta^\dagger Y_\eta + Y_\eta Y_\phi^\dagger Y_\phi,$$

$$16\pi^2\beta_{Y_e} = \beta_{Y_e}^{\mathrm{SM}} + \frac{1}{2}(Y_N^\dagger Y_N + \mathrm{Tr}Y_N^* Y_N^T)Y_e - \frac{3}{2}Y_N Y_N^\dagger Y_e,$$
$$16\pi^2\beta_{Y_d} = \beta_{Y_d}^{\mathrm{SM}} + \frac{1}{2}(Y_N^\dagger Y_N + \mathrm{Tr}Y_N^* Y_N^T)Y_d,$$
$$16\pi^2\beta_{Y_u} = \beta_{Y_u}^{\mathrm{SM}} + \frac{1}{2}(Y_N^\dagger Y_N + \mathrm{Tr}Y_N^* Y_N^T)Y_u$$

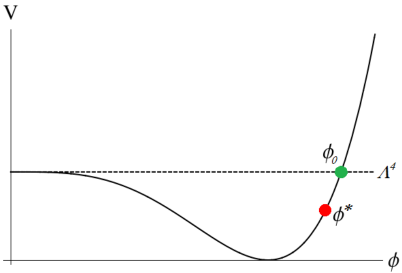
Model A: M_P by hand. More Results



■ BICEP2 & Planck 2σ region ■ chaotic ■ hilltop

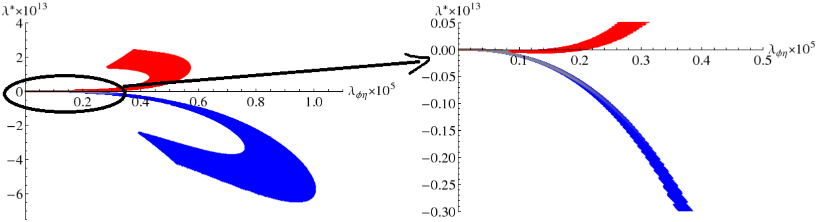
- $m_\phi \sim 10^{13}$ GeV
- hilltop $\rightarrow \lambda(\phi)^* < 0$. In the ■ region: $0 > \lambda(\phi)^* > -2\times10^{-14}$
- chaotic $\rightarrow$ usually $\lambda(\phi)^* > 0$, but in the ■ region: $0 > \lambda(\phi)^* > -3\times10^{-16}$

Model A: M_P by hand. More on $\lambda(\phi)^* < 0$



- $N(\phi^*) = 60$
- $V(\phi_0) = \Lambda^4$
- $\lambda_\phi \simeq \beta_{\lambda_\phi} \ln \frac{\phi}{\phi_0} \rightarrow \begin{cases} \lambda_\phi(\phi > \phi_0) > 0 \\ \lambda_\phi(\phi < \phi_0) < 0 \end{cases}$

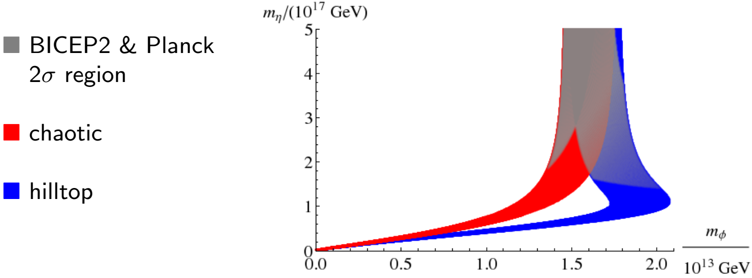
Model A: M_P by hand. More Results



■ BICEP2 & Planck 2σ region ■ hilltop ■ chaotic

- $\lambda_{\phi\eta} \lesssim 10^{-5}$
- $\lambda_{\phi\eta}$ is small enough to be treated as a constant and so $\beta_{\lambda_\phi} = \frac{\lambda_{\phi\eta}^2}{32\pi^2}$.

Model A: M_P by hand. More Results



- For consistency, $\lambda_\eta > 0$, but otherwise can have any value.
- $m_\eta = \sqrt{\frac{\lambda_{\phi\eta}}{2}} v_\phi$
- $m_\eta \sim 10^{17}$ GeV, $m_\eta > V^* \sim 10^{16}$ GeV, and $m_\eta \gg m_\phi$
  $\Rightarrow \eta$ is decoupled and is frozen at its minimum $\eta = 0$ during inflation.
- Lebedev and Westphal, Phys.Lett. B719 (2013) 415-418: $\eta = 0$ is quickly achieved during inflation due to $\lambda_{\phi\eta} \Rightarrow$ model is self-consistent.

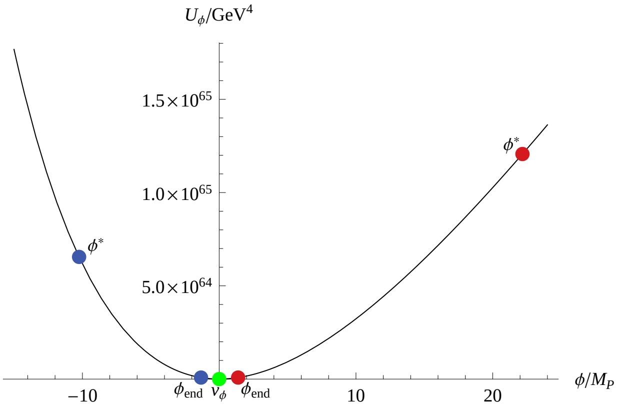
Model B: Induced M_P and ξ. Simplify $\lambda_\phi(\phi)$

- $\lambda_\phi(\phi) \simeq \frac{1}{2}\lambda''_\phi(v_\phi)(\phi - v_\phi)^2$
- The β-functions are logarithmic derivatives of couplings: $\beta_{\lambda_i} = \mu\frac{d\lambda_i}{d\mu}$. Thus, using $\mu = \phi$

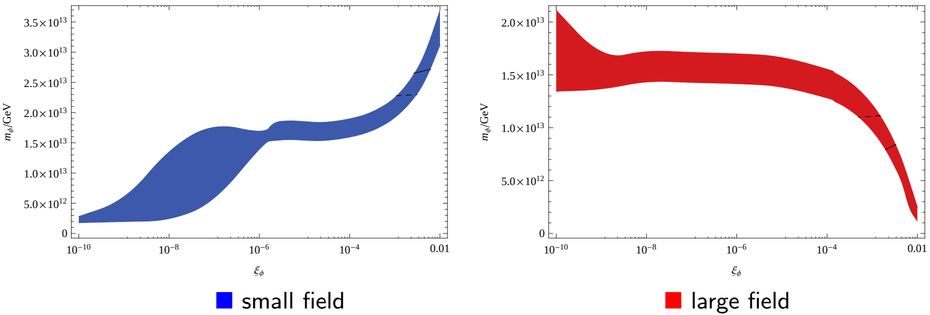
$$\lambda''_\phi = \frac{d}{d\mu}\left(\frac{\beta_{\lambda_\phi}}{\mu}\right) = \frac{1}{\mu}\frac{d\beta_{\lambda_\phi}}{d\mu} - \frac{\beta_{\lambda_\phi}}{\mu^2}$$

- $\beta_{\lambda_\phi}(v_\phi) = \mu\frac{d\lambda_\phi}{d\mu}(v_\phi) = 0$
- $\lambda''_\phi(v_\phi) = \frac{\beta'_{\lambda_\phi}(v_\phi)}{v_\phi} = \cdots = \frac{\xi_\phi\lambda_{\phi\eta}\left[12\lambda_\eta+(8-3\sqrt{2})\lambda_{\phi\eta}-48(1+\sqrt{2})y_\eta^2\right]}{512\pi^4 M_P^2}$
- N.B. the numerator has to be positive

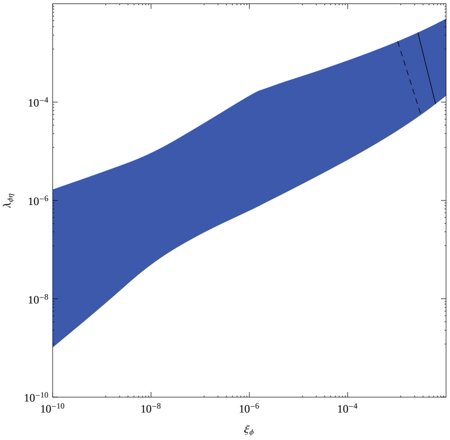
U_ϕ around v_ϕ



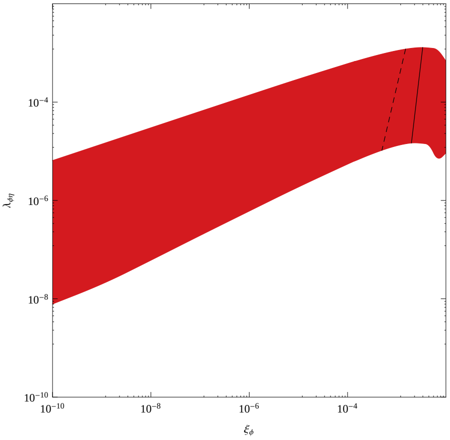
m_ϕ vs ξ_ϕ



$\lambda_{\phi\eta}$ vs ξ_ϕ

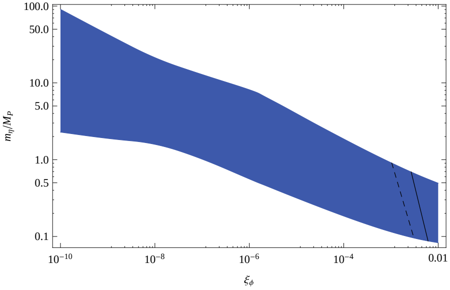


small field

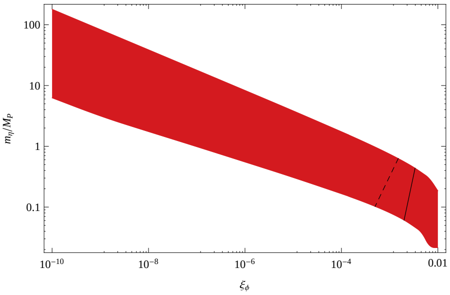


large field

m_η vs ξ_ϕ

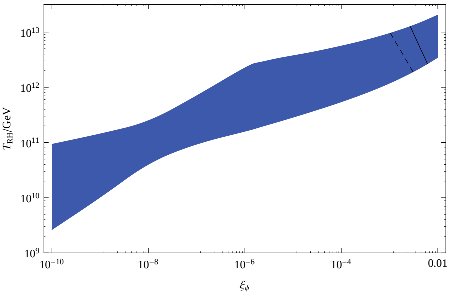


small field

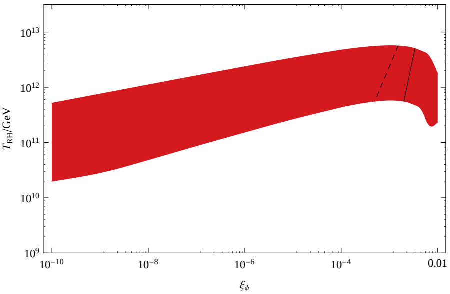


large field

T_{RH} vs ξ_ϕ



small field



large field