Applications of (next-to) soft radiation

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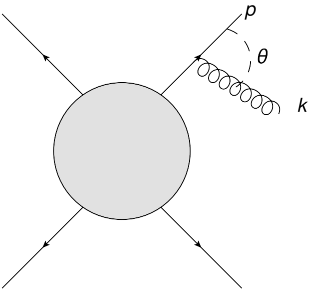
NExT Workshop at QMUL

Overview

- Brief introduction to (next-to-) soft divergences.
- Applications to collider physics.
- Applications to gravity.
- Outlook.

- In scattering amplitudes, get IR singularities due to soft or collinear gluon emissions:

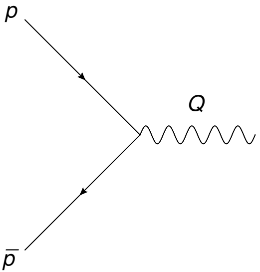


$$\frac{1}{p \cdot k} = \frac{1}{|\boldsymbol{p}||\boldsymbol{k}|(1 - \cos\theta)}.$$

- Formal singularity cancels upon combining real and virtual (Block, Nordsieck).

- Soft / collinear radiation is *universal*.
- Physics: it has an infinite wavelength, so cannot resolve the underlying amplitude.

- A major application of these ideas is in collider physics.
- Although IR singularities cancel, they leave behind large contributions to perturbative quantities.
- Consider e.g. the production of a vector boson at a collider ("Drell-Yan production"):



- Let $z = Q^2/s$ be the fraction of (squared) energy s carried by the vector boson.
- At LO, $z = 1$, and thus the cross-section is

$$\frac{d\sigma^{(0)}}{dz} \propto \delta(1-z).$$

- At next-to-leading order (NLO), radiation can carry energy, so that
$$0 \leq z \leq 1.$$
- The NLO cross-section then turns out to be
$$\frac{d\sigma^{(1)}_{q\bar{q}}}{dz} \sim \frac{\alpha_s}{2\pi}\left[4(1+z^2)\left(\frac{\ln(1-z)}{1-z}\right)_+ - 2\frac{1+z^2}{1-z}\ln(z) \right.$$
$$\left. +\delta(1-z)\left(\frac{2\pi^2}{3}-8\right)\right].$$
- It contains highly divergent terms as $z \to 1$.
- Looks like perturbation theory is in trouble!
- Let's go one order higher and see what happens...

- At NNLO the problem is even worse! One has

$$\frac{d\sigma^{(2)}_{q\bar{q}}}{dz} \sim C_F^2 \left(\frac{\alpha_s}{2\pi}\right)^2 \left[128 \left(\frac{\ln^3(1-z)}{1-z}\right)_+ - 256 \left(\frac{\ln(1-z)}{1-z}\right)_+ + \ldots\right],$$

  where $\ldots$ denotes terms suppressed by $(1-z)$.
- Logs get higher at higher orders in perturbation theory...
- ... which indeed breaks down as $z \to 1$.
- Precisely the regime where the vector boson is produced near threshold, so that extra radiation is soft / collinear!
- The problem terms are echoes of the cancelled IR divergences.
- Thus, this problem affects many different scattering processes...

- For heavy particles produced near threshold, we can define a ξ, where $\xi \to 0$ at threshold (e.g. $\xi = (1 - z)$).
- Then the general structure of any such cross-section is:

$$\frac{d\sigma}{d\xi} = \sum_{n,m} \alpha^n \left[c_{nm}^{(0)} \left(\frac{\ln^m \xi}{\xi} \right)_+ + c_{nm}^{(1)} \ln^m \xi + \ldots \right].$$

- First set of terms correspond to (leading) threshold logs: pure soft and / or collinear.
- Second set of terms is next-to-leading power (NLP) threshold logs: next-to-soft and / or collinear.
- For $\xi \to 0$, we need to rethink perturbation theory.

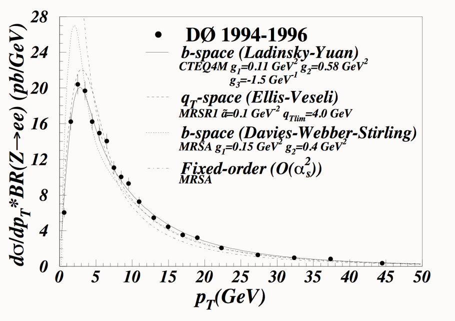
- The solution to this problem is to somehow work out what the large logs are to all orders in α_s.
- Then we can sum them up to get a function of α_s that is better behaved than any fixed order perturbation expansion.
- Toy example: consider the function

$$e^{-\alpha_s x} = \sum_{n=0}^{\infty} \frac{\alpha_s^n (-x)^n}{n!}.$$

- Each term diverges as $x \to \infty$, but the all-order result is well-behaved.

What is resummation good for?

- There are hundreds of collider observables for which resummation is **crucial**!



1. **Total cross-sections**: More accurate central value, reduced theoretical (scale) uncertainty.
2. **Differential cross-sections**: correct shape, less uncertainty.

- Even without resummation, can use knowledge of threshold logs to get approiximate higher order cross-sections (e.g. top pair, single top, DY, Higgs...).

- Many approaches exist for resumming leading threshold logs.
- Original diagrammatic approaches by e.g. Sterman; Catani, Trentadue.
- Can also use Wilson lines (Korchemsky, Marchesini), or the renormalisation group (Forte, Ridolfi).
- A widely used approach is to treat soft and collinear gluons as separate fields in an effective theory: soft-collinear effective theory (SCET) (Becher, Neubert; Schwartz; Stewart).
- All approaches have the *factorisation* of soft / collinear physics at their heart.

- The general structure of an n-point amplitude is

$$\mathcal{A}_n = \mathcal{H}_n \times \mathcal{S} \times \frac{\prod_i J_i}{\prod_i \mathcal{J}_i}.$$

- Here $\mathcal{H}_n$ is the *hard function*, and is IR finite.
- The *soft* and *jet functions* $\mathcal{S}$ and J_i collect soft / collinear singularities respectively.
- The *eikonal jets* $\mathcal{J}_i$ remove any double counting.
- Soft and jet functions are *universal*: once they are calculated, can resum logs in *any* process.
- This formula works at leading power (LP) only.

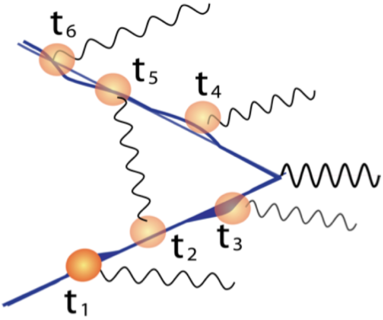
- To date, much less has been known about NLP effects.
- Known for a while to be numerically significant e.g. in Higgs production (Kramer Laenen, Spira; Harlander, Kilgore; Catani, de Florian, Grazzini, Nason).
- This has been confirmed by recent N^{3}LO Higgs results (Anastasiou, Duhr, Dulat, Herzog, Mistlberger).
- There are three good reasons to study NLP logs:
  1. Resummation of them will improve precision.
  2. Even without resummation, NLP logs may provide good approximate N^{n}LO cross-sections.
  3. Can improve the stability of numerical codes.

- Next-to-soft effects in particular scattering processes classified to all orders by (Almasy, Moch, Presti, Soar, Vermaseren, Vogt).
- Can also be classified using the *method of regions* (Beneke, Smirnov, Pak, Jantzen) (studies by Bahjat-Abbas, Bonocore, Sinninghe Damsté, Laenen, Magnea, Vernazza, White).
- None of the previous approaches is fully general - but strong hints of an underlying structure.
- Can we predict NLP logs in an arbitrary process?
- Can they be written in terms of universal functions (like LP effects)?
- Encouraging recent progress...

- It is well-known that LP effects can be described using *Soft-Collinear Effective Theory* SCET (Stewart, Schwartz, Bauer, Fleming; Becher, Neubert).
- The same language can be extended to NLP level (Beneke, Campanario, Mannel, Pecjak;Larkoski, Neill, Stewart).
- Phenomenology explored by Feige, Kolodrubetz, Moult, Stewart, Rothen, Tackmann, Zhu; Boughezal, Liu, Petriello.
- Resummation of event shapes (Stewart et. al.)...
- ...and Drell-Yan (Beneke, Broggio, Garny, Jaskiewicz, Szafron, Vernazza, Wang).

- The soft-collinear factorisation formula can be generalised to next-to-leading power level (Bonocore, Laenen, Magnea, Melville, Vernazza, White).
- A new quantity appears at next-to-soft level: the *jet emission function*.
- Calculated for quarks to one-loop order, but more functions needed (also for gluons) (Gervais).
- For the highest power of the NLP log, can resum using path integral methods...

- Next-to-soft effects for massive particles considered using worldline methods by Laenen, Stavenga, White (2008).



- Can replace propagators for external legs by quantum mechanics path integrals.
- Leading term in perturbative expansion is classical trajectory (soft limit).
- First-order wobbles give next-to-soft behaviour.

- Also works for gravity (White, 2011).

- The path integral method can be used to define a universal *next-to-soft function*, whose Feynman diagrams are straightforward.
- It captures the dominant NLP terms at all orders in perturbation theory...
- ...and tells us how to sum them up!
- Can resum total cross-sections for arbitrary $q\bar{q}$ or gg initial states, including Drell-Yan, Higgs, Higgs pair, W and Z pairs...
- Paper in progress (Bahjat-Abbas, Bonocore, Laenen, Magnea, Sinninghe Damsté, Vernazza, White).

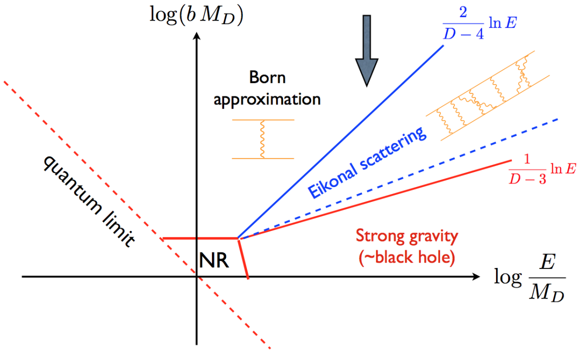
Applications at fixed order

- Even if we don't sum up NLP logs, we can still use them to get approximate higher order cross-sections.
- One can derive universal forms for NLO cross-sections with colour-singlet final states (Del Duca, Laenen, Magnea, Vernazza, White)...
- ...or even states with multiple final state coloured particles (van Beekveld, Beenakker, Laenen, White).
- Numerical studies of where these effects are important are in progress in prompt photon production (van Beekveld, Beenakker, Basu, Laenen, Misra, Motylinski).

Are there any other processes people want us to look at?

- Remarkably, the structure of next-to-soft corrections appears to be similar in QCD and gravity.
- This may be related to the *BCJ double copy*, a recently conjectured relationship between the two theories (Bern, Carrasco, Johansson).
- QCD next-to-soft effects are important for collider physics...
- ...but there may also be phenomenological consequences of next-to-soft graviton emission!
- Consider two massive particles scattering off each other...

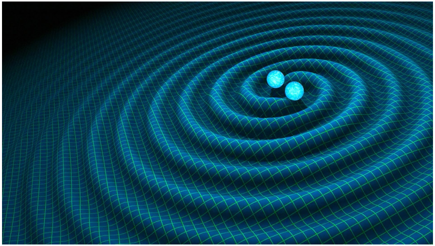
- Soft gravitons dominate in the high-energy (Regge) limit.



- Different regions in energy / impact parameter (Giddings, Schmidt-Sommerfeld, Andersen).
- Next-to-soft corrections probe unknown parts of this diagram.

- Large amount of work at leading soft level from the 1990s onwards (Amati, Ciafaloni, Veneziano, Colferai, Falcioni; 't Hooft; Verlinde^2; Jackiw, Kabat, Ortiz).
- Recently, people have used QCD-like language to extend this to next-to-soft level (Akhoury, Saotome, Sterman; Melville, Naculich, Schnitzer, White).
- Methods from string theory also help (Koemans Collado, di Vecchia, Russo, Thomas).
- May lead to new conceptual insights regarding gravity above the Planck scale...
- ...but may also have more practical applications.

- Gravitational waves have been recently discovered by LIGO.



- Spectra are difficult to calculate in General Relativity.
- Methods from scattering amplitudes may help.

- An open problem is how to extract classical GR results from field theory Feynman integrals.
- This is directly related to the (next-to) soft expansion!

- Soft radiation gives large contributions in perturbation theory, that need to be resummed.
- We are starting to get results beyond the leading soft approximation.
- Results can increase the precision of (total or differential) cross-sections at the LHC.
- Similar physics underlies open problems in gravity!

- How do we completey classify next-to-leading power threshold logs?
- Where are these effects important?
- Can they be implemented in numerical codes for straightforward implementation in experimental analyses?
- Can we understand (quantum) gravity at high energies using next-to-soft physics?
- Can we improve predictions for classical GR using QCD-like methods?