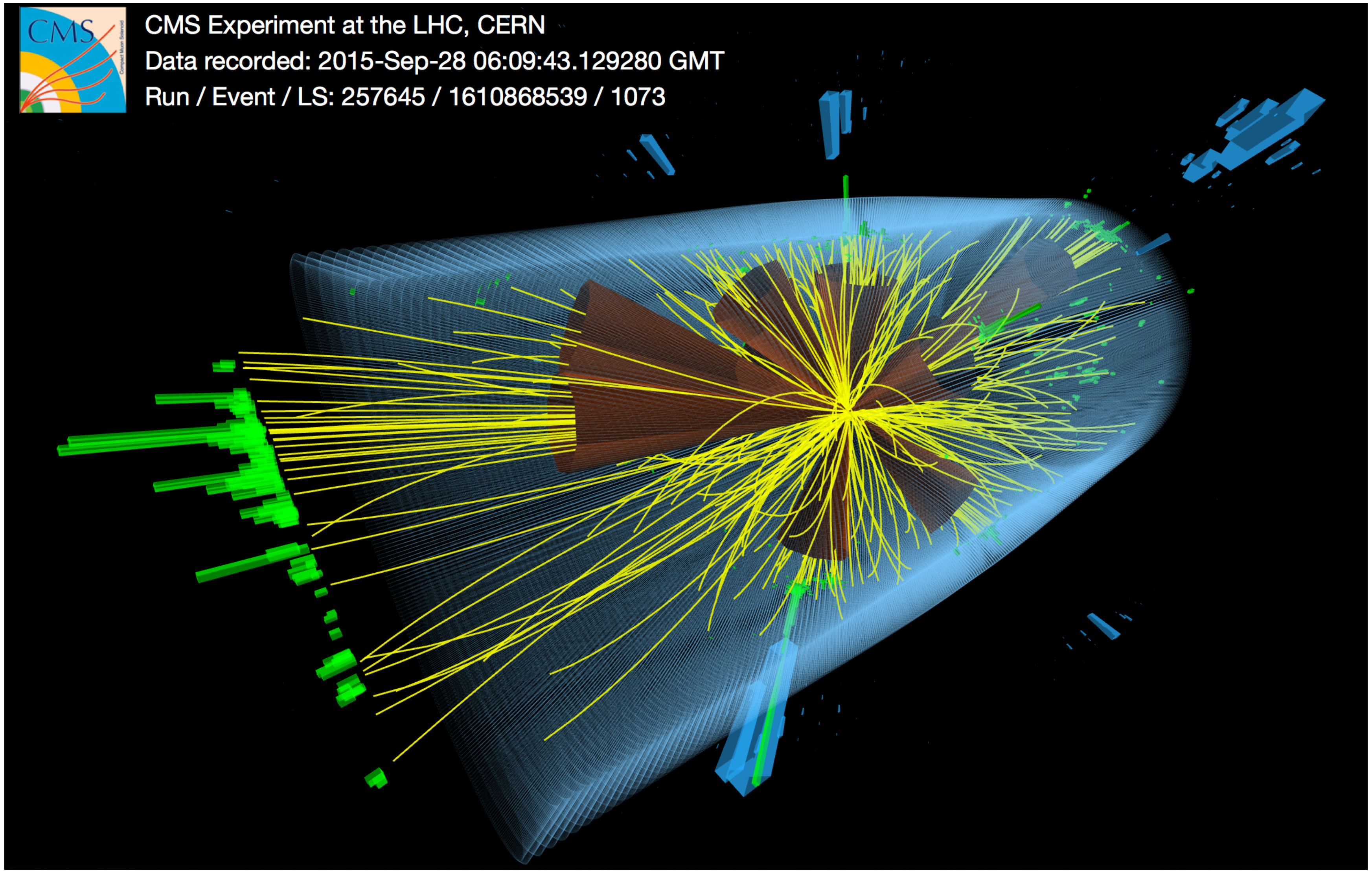
MultiJet Predictions for the LHC

Jenni Smillie

Higgs Centre for Theoretical Physics

Prelude

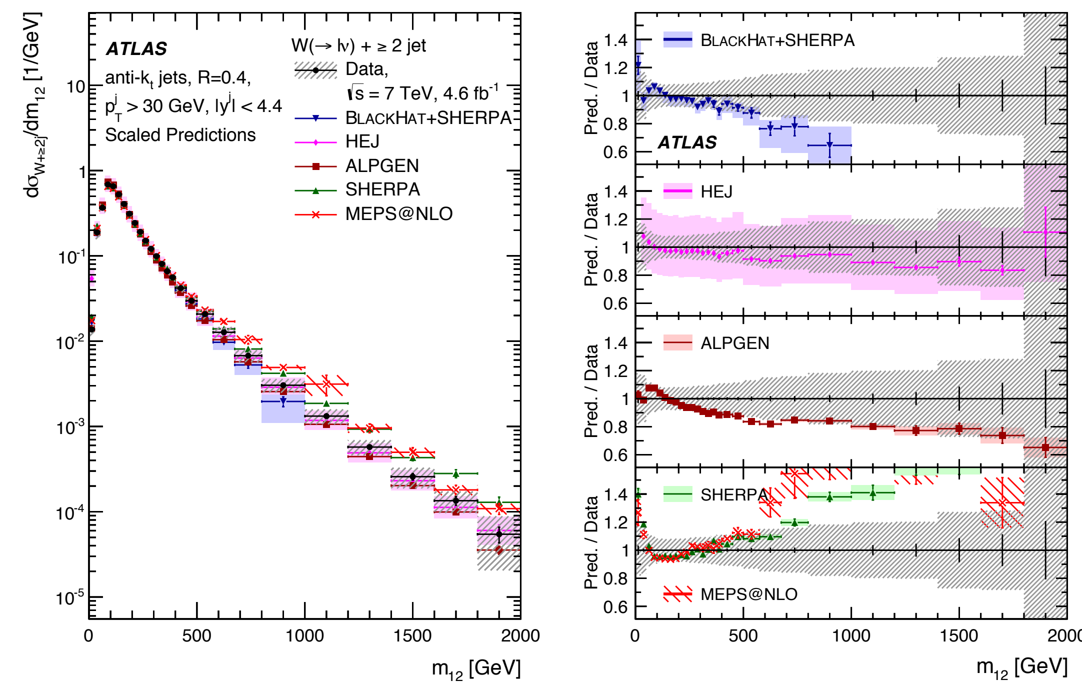


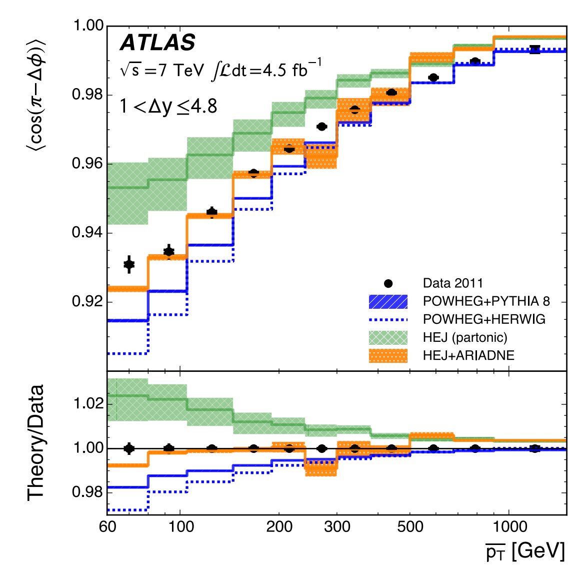
12 jets with p_T > **50 GeV** at CMS (13 TeV)

Just one sign of importance of higher-order terms in α_s at large s

Outline

- Motivation
- High Energy Limit (improved)
- High Energy Jets
- Comparisons to Data
- NEW Higgs Boson plus Dijets





QCD at High Energy

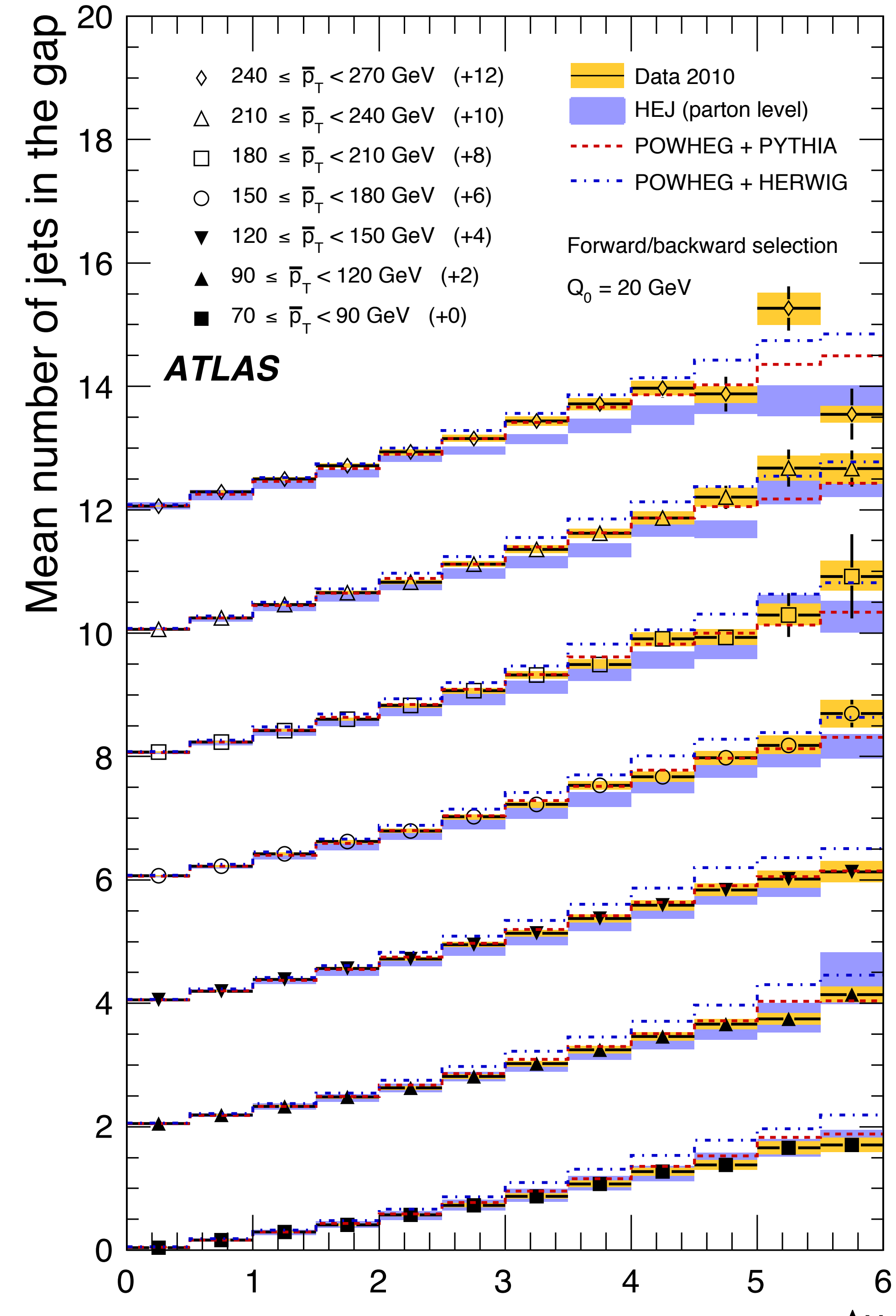
- Already at 7 TeV, (n+1)-jet rates are not small compared to n-jet rates [0.2 rising to 0.3 after VBF cuts]
- Stability associated with NLO fails in difficult regions of phase space
- Extra power of α_s compensated by large real-emission phase space and large logarithms - **especially at 13 TeV, 100 TeV…**
- Large rapidity separations or large invariant mass enhance (multi-)jet production (e.g. VBF)

Higgs boson analyses and searches for new physics put us right into the most difficult regions

QCD at High Energy

Plot shows average number of additional jets (above 2) at 7 TeV in slices of p_T

More than one extra jet for Δy > 3
Manifestly beyond NLO!

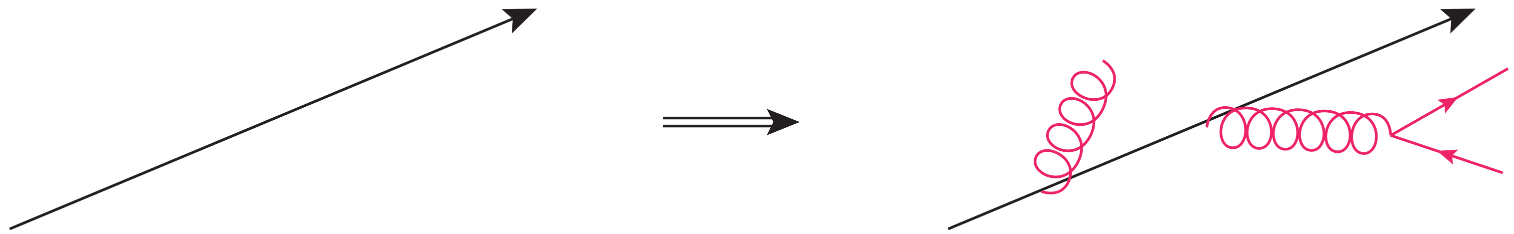


arXiv:1107.1641

QCD at all orders

- Have been adding all-order corrections via parton showers since '80s **Pythia, Herwig, Sherpa (later) …**

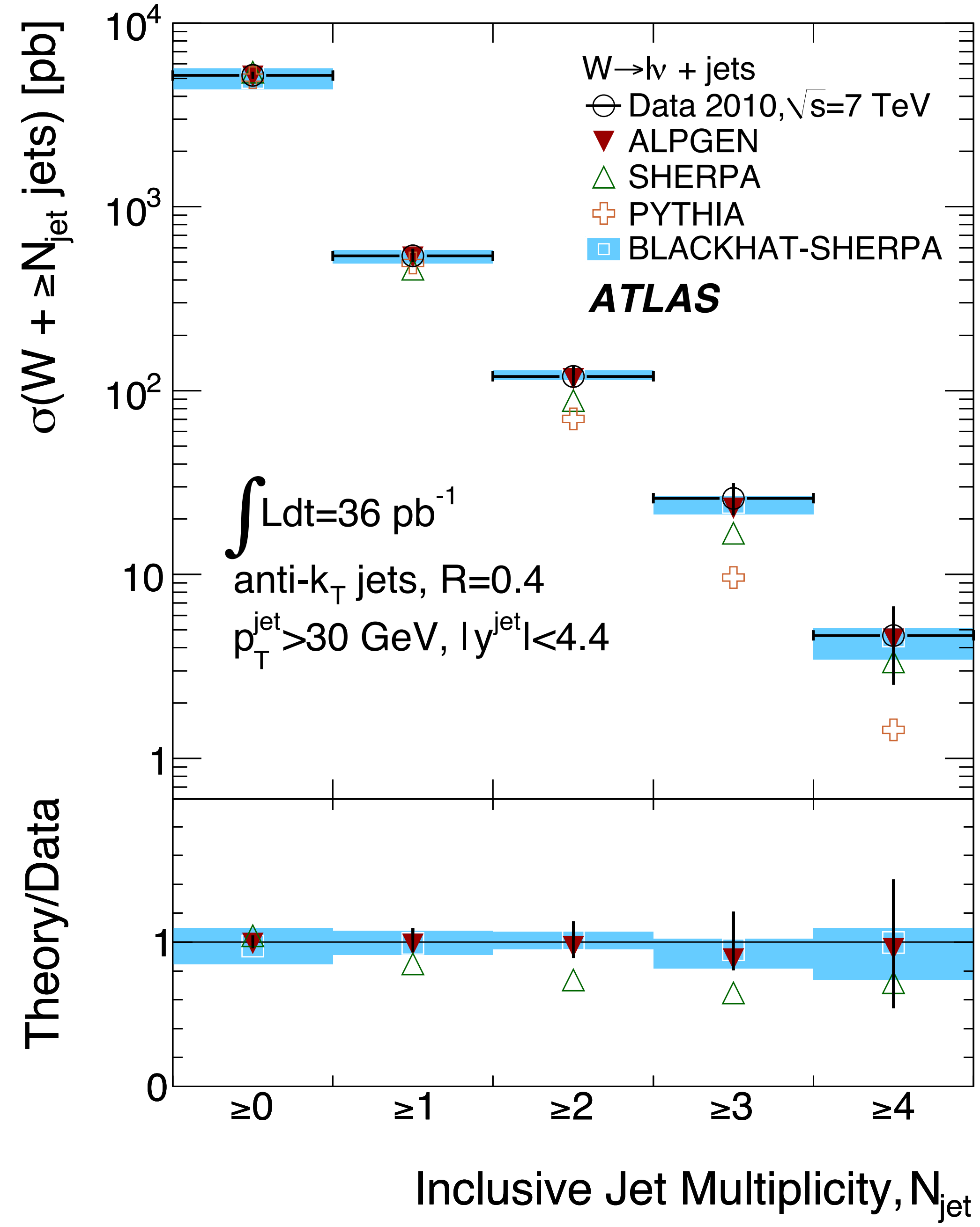
- These capture terms which are enhanced at small s_{ij} = soft and collinear emissions



- Now largely automated with NLO matrix elements
PowhegBox, Madgraph5_aMC@NLO, Sherpa/OpenLoops

But…

- This is not designed to describe additional hard, wide-angle radiation
- Smart merging approaches help a bit
  Sherpa MEPS(@NLO), MENLOPS; UNLOPS, Plätzer, …
- There is a perturbative instability: needs an all-order approach to hard, wide-angle radiation



ATLAS W+Jets (2010 data) arXiv:1201.1276

Which "all-order"?

$$|M_{2j}|^2 = \alpha_s^2\left(a_2(\hat{s}^2/\hat{t}^2) + b_2\right)$$
$$+\alpha_s^3\left(a_3(\hat{s}^2/\hat{t}^2)\log(\hat{s}/\hat{t}) + b_3(\hat{s}^2/\hat{t}^2) + c_3\right)$$
$$+\alpha_s^4\left(a_4(\hat{s}^2/\hat{t}^2)\log^2(\hat{s}/\hat{t}) + b_4(\hat{s}^2/\hat{t}^2)\log(\hat{s}/\hat{t}) + \ldots\right)$$
$$+\ldots$$

- LO = first line
- NLO = first two lines
- Leading logs = **the 'a'-terms**

Which "all-order"?

$$|M_{2j}|^2 = \alpha_s^2 \left(a_2(\hat{s}^2/\hat{t}^2) + b_2\right)$$
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$$+ \alpha_s^4 \left(a_4(\hat{s}^2/\hat{t}^2)\log^2(\hat{s}/\hat{t}) + b_4(\hat{s}^2/\hat{t}^2)\log(\hat{s}/\hat{t}) + \ldots\right)$$
$$+ \ldots$$

- LO = first line
- NLO = first two lines
- Leading logs = the 'a'-terms
- Our description = **LO + LL** (plus NLO cross section, for now…)

Regge Theory

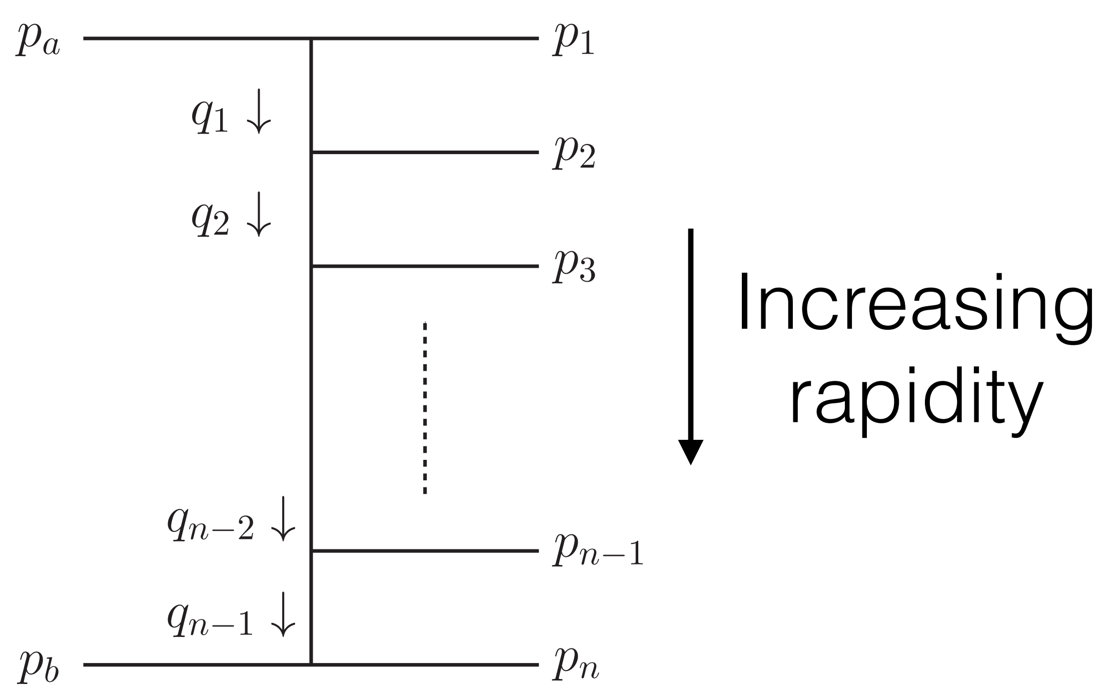
The leading (highest-power) logarithms arise in $2 \to n$ scattering in the High Energy or Multi-Regge Kinematic (MRK) limit:

$$s_{ij} \to \infty, \qquad p_{\perp i} \text{ finite} \qquad i,j = 1,\ldots,n$$

This is equivalent to all particles being well-separated in rapidity.

Ordering outgoing particles in rapidity defines an effective t-channel:

$$q_1 = p_a - p_1, \quad q_i = q_{i-1} - p_i$$
$$t_i = q_i^2$$



Regge Theory

Then Regge Theory tells us the amplitudes scale in MRK as

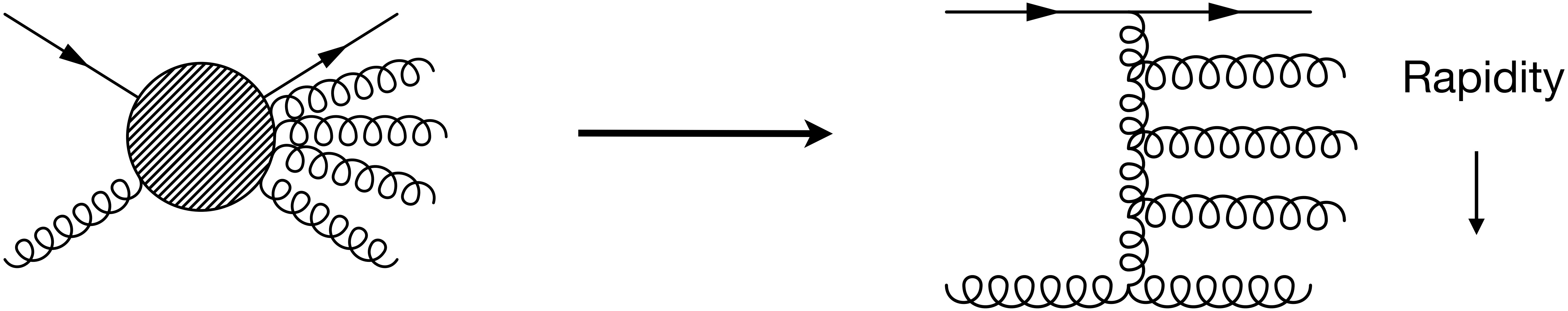
$$\mathcal{M} \sim s_{12}^{\alpha_1(t_1)} \dots s_{n-1,n}^{\alpha_{n-1}(t_{n-1})} \; \gamma(\{p_{\perp,i}\}, \dots)$$

transverse (finite) scales

α_i is the spin of the particle in that t-channel

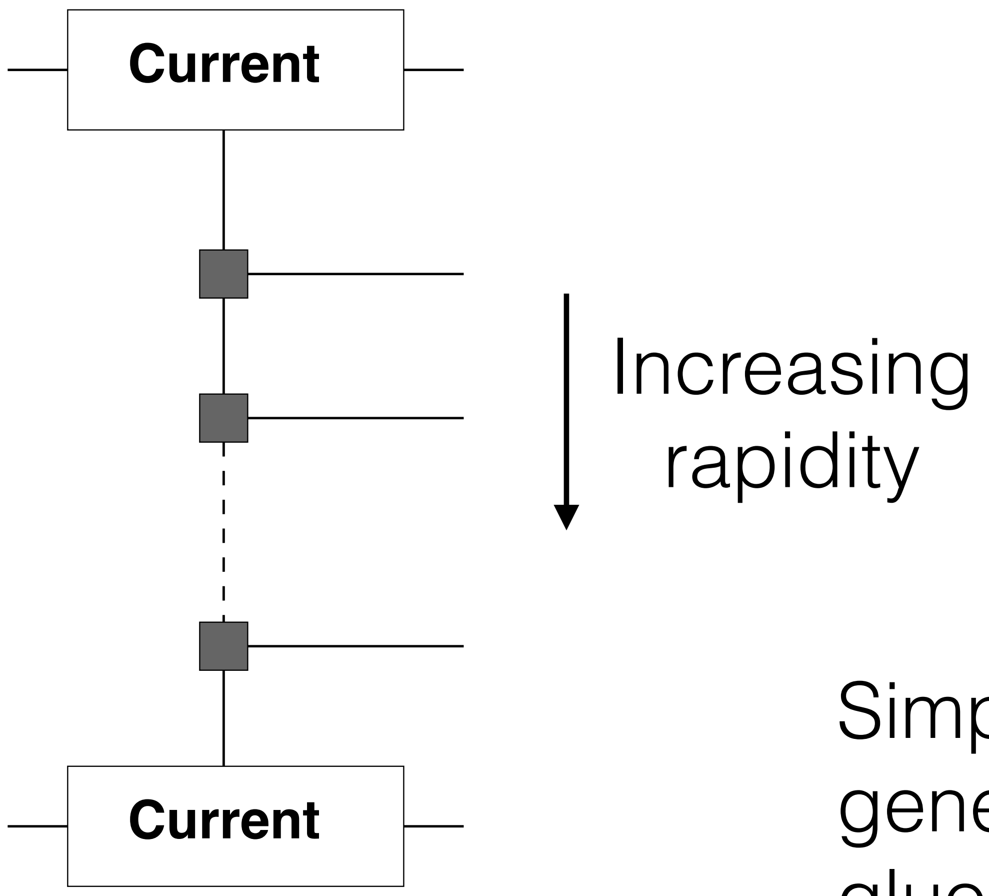
Brower, DeTar, Weis 1974; Fadin, Fiore, Kozlov, Reznichenko hep-ph/0602006

Therefore, in QCD, processes dominated by gluon-exchange…



… and by subprocesses which allow maximum gluon exchanges

Regge Theory



Further, in the MRK limit the amplitudes factorise into independent pieces

see Duhr, Liu arXiv:1811.06478 for related work from scattering equations

Simpler structure allows for an efficient event generator for arbitrary numbers of quarks/gluons. Applies to loop diagrams too (needed to regulate soft).

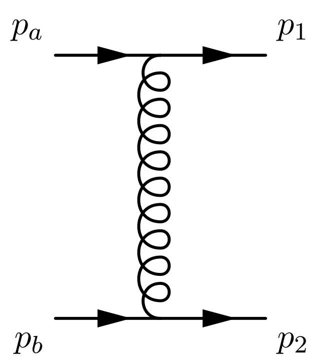
SEE LATER

HEJ2 event generator: http://hej.web.cern.ch

Pieces I: Currents

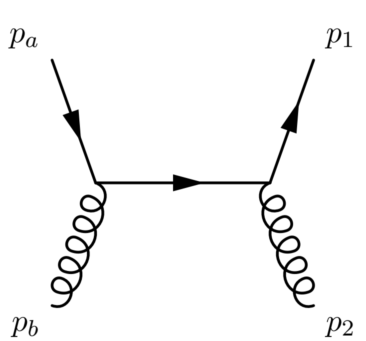
Matrix element pieces independent of the rest of the chain - pick convenient processes to derive them

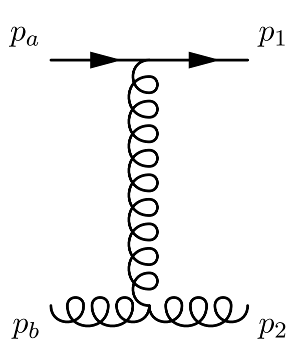
- Incoming quarks: straight-forward

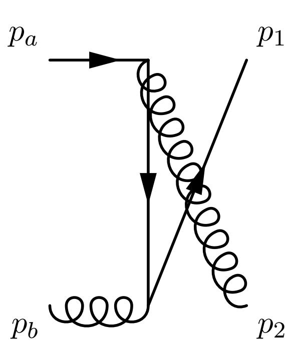


$$\frac{8g_s^4}{9}\frac{|j^\mu(p_a,p_1)\cdot j_\mu(p_b,p_2)|^2}{\hat{t}^2}$$
$$=\frac{4g_s^4}{9}\frac{\hat{s}^2+\hat{u}^2}{\hat{t}^2}$$

- Incoming gluons: surprisingly so!







- Exact result: $\frac{g_s^4 C_{CAM}}{6}\;\frac{|j^\mu(p_a,p_1)\cdot j_\mu(p_b,p_2)|^2}{\hat{t}^2}$

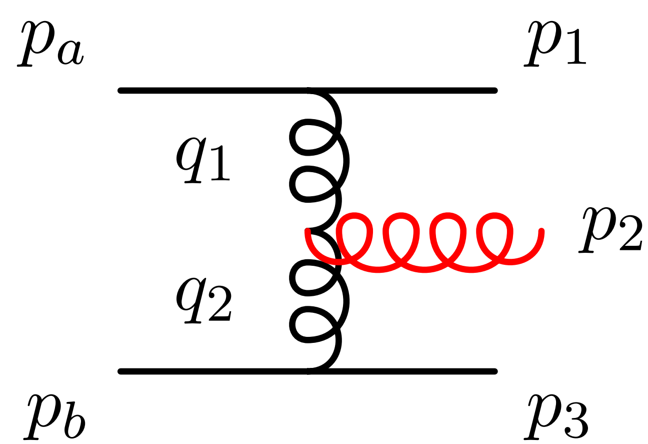
  with $C_{CAM}=\frac{1}{2}\left(C_A-\frac{1}{C_A}\right)\left(\frac{p_b^-}{p_2^-}+\frac{p_2^-}{p_b^-}\right)+\frac{1}{C_A}$

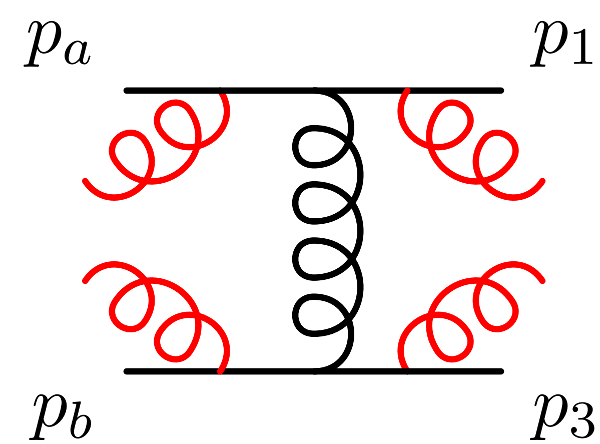
- Only t-pole remains explicitly

Andersen & JMS arXiv:0910.5113

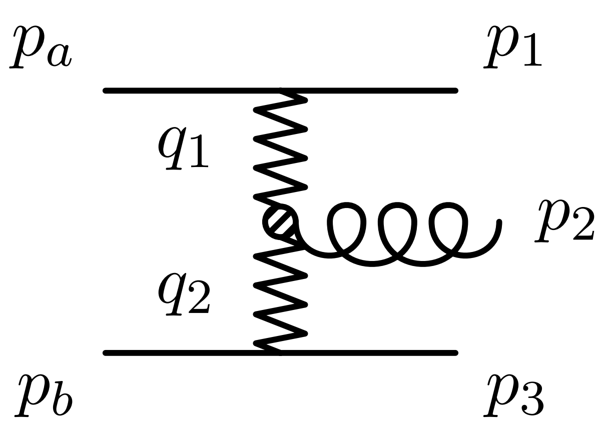
Pieces II: Vertices

- Use qQ → qgQ





- In HE limit, colour factors combine to give



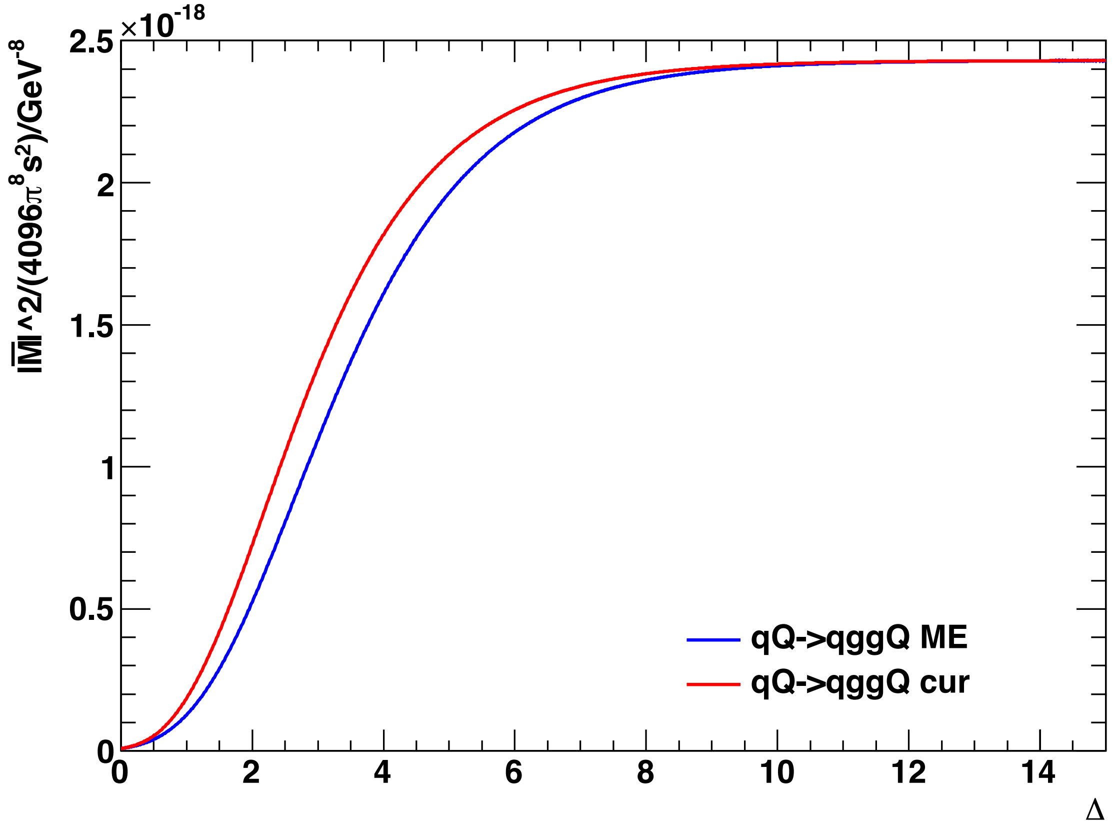
$$\mathcal{A}_{qQ\to qgQ} = g_s^3\, \mathcal{C}_g\, \varepsilon_\rho^*\, \frac{j^\mu(p_a,p_1)\cdot j_\mu(p_b,p_3)}{q_1^2 q_2^2}\, V^\rho(q_1,q_2)$$

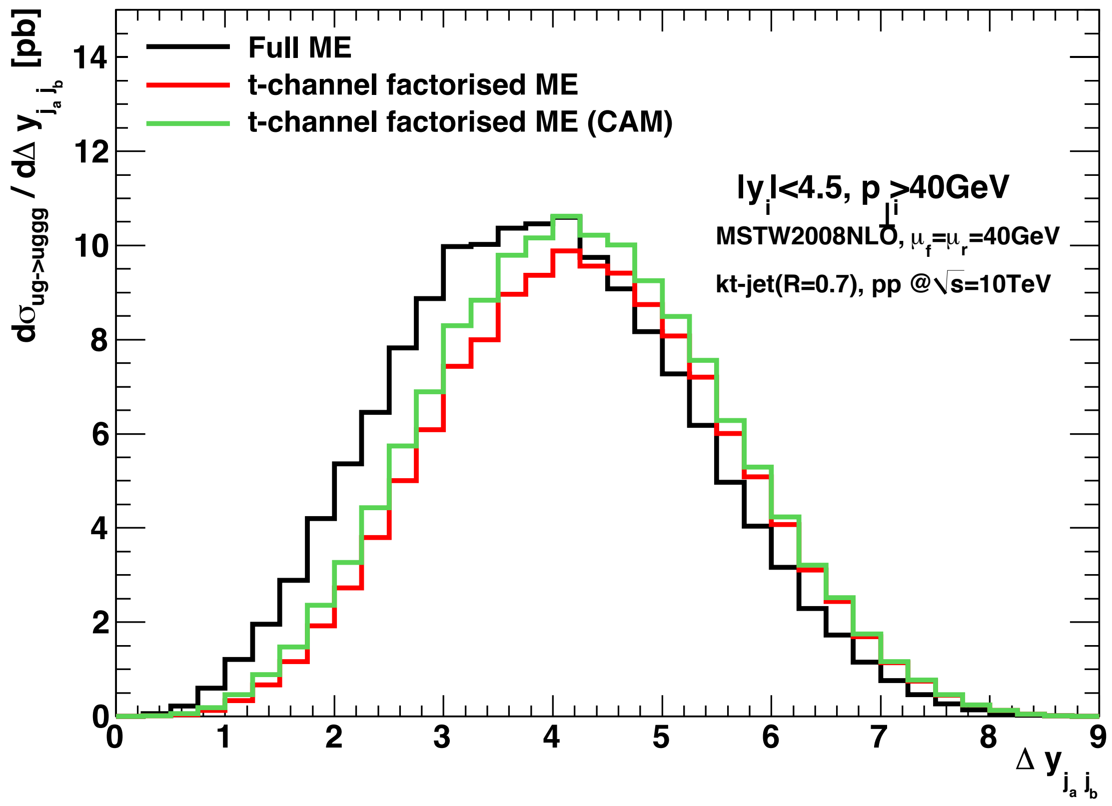
$$\begin{aligned} V^\rho(q_1,q_2) = & -(q_1+q_2)^\rho \\ & + \frac{p_a^\rho}{2}\left(\frac{q_1^2}{p_2\cdot p_a} + \frac{p_2\cdot p_b}{p_a\cdot p_b} + \frac{p_2\cdot p_3}{p_a\cdot p_3}\right) + p_a \leftrightarrow p_1 \\ & - \frac{p_b^\rho}{2}\left(\frac{q_2^2}{p_2\cdot p_b} + \frac{p_2\cdot p_a}{p_b\cdot p_a} + \frac{p_2\cdot p_1}{p_b\cdot p_1}\right) - p_b \leftrightarrow p_3. \end{aligned}$$

- Gauge invariant in *all* of phase space

Andersen & JMS arXiv:0908.2786

How good is it?



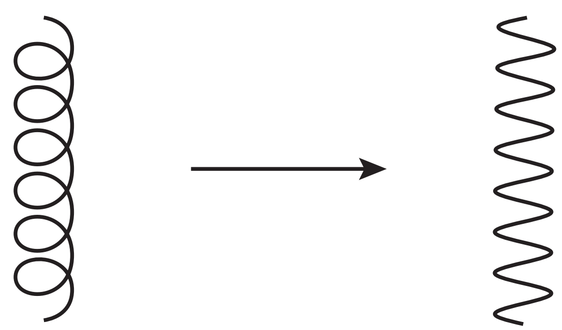


$qQ \to qggQ$ $\qquad$ $qg \to qggg$

Pieces III: Regularisation

Remains to regulate divergences as $p_i \to 0$

HE Limit of virtual corrections given by **Lipatov ansatz**



$$= \frac{1}{t_i} \exp[\hat{\alpha}(q_i)(y_{i-1} - y_i)]$$

$$\hat{\alpha}(q_i) = \alpha_s \; C_A \; t_i \int \frac{d^{2+2\epsilon} k_\perp}{(2\pi)^{2+2\epsilon}} \frac{1}{k_\perp^2 \, (q_i - k)_\perp^2}$$

$$\to -g_s^2 C_A \; \frac{\Gamma(1-\varepsilon)}{(4\pi)^{2+\varepsilon}} \frac{2}{\varepsilon} \left(\mathbf{q}^2/\mu^2\right)^{\varepsilon}$$

Proved to next-to-leading log

Fadin, Fiore, Kozlov & Reznichenko: hep-ph/0602006

Main Equations

Squared Matrix Element

$$\overline{\left|\mathcal{M}^{\text{reg}}_{\text{HEJ}}(\{p_i\})\right|}^2 = \frac{1}{4\,(N_C^2-1)}\ \left\|S_{f_1 f_2\to f_1 f_2}\right\|^2$$
$$\cdot\ \left(g^2\ K_{f_1}\ \frac{1}{t_1}\right)\cdot\ \left(g^2\ K_{f_2}\ \frac{1}{t_{n-1}}\right)$$
$$\cdot\prod_{i=1}^{n-2}\left(g^2 C_A\ \left(\frac{-1}{t_i t_{i+1}}V^\mu(q_i,q_{i+1})V_\mu(q_i,q_{i+1}) - \frac{4}{\mathbf{p}_i^2}\ \theta\left(\mathbf{p}_i^2<\lambda^2\right)\right)\right)$$
$$\cdot\prod_{j=1}^{n-1}\exp\left[\omega^0(q_j,\lambda)(y_j-y_{j+1})\right],$$

Cross Section

$$\sigma_{2j}^{\text{resum,match}} = \sum_{f_1,f_2}\ \sum_{n=2}^{\infty}\ \prod_{i=1}^{n}\left(\int_{p_{i\perp}=\lambda}^{p_{i\perp}=\infty}\frac{\mathrm{d}^2\mathbf{p}_{i\perp}}{(2\pi)^3}\ \int\frac{\mathrm{d}y_i}{2}\right)\ \frac{\overline{\left|\mathcal{M}_{\text{HEJ}}^{f_1 f_2\to f_1 g\cdots g f_2}(\{p_i\})\right|}^2}{\hat{s}^2}$$
$$\times\ \sum_m \mathcal{O}^e_{mj}(\{p_i\})\ w_{m-\text{jet}}$$
$$\times\ \ x_a f_{A,f_1}(x_a,Q_a)\ x_2 f_{B,f_2}(x_b,Q_b)\ (2\pi)^4\ \delta^2\!\left(\sum_{i=1}^{n}\mathbf{p}_{i\perp}\right)\ \mathcal{O}_{2j}(\{p_i\})$$

$$\sigma_{2j} = \sigma_{2j}^{\text{resum,match}} + \sum_n \sigma_{nj}^{\text{non}-\text{FKL}}$$

Andersen & JMS arXiv:0908.2786, 0910.5113, 1101.5394

Main Equations

$$\overline{\left|\mathcal{M}^{\rm reg}_{\rm HEJ}(\{p_i\})\right|}^2 = \frac{1}{4\,(N_C^2-1)}\,\left\|S_{f_1 f_2\to f_1 f_2}\right\|^2 \cdot \left(g^2\,K_{f_1}\,\frac{1}{t_1}\right)\cdot\left(g^2\,K_{f_2}\,\frac{1}{t_{n-1}}\right) \cdot \prod_{i=1}^{n-2}\left(g^2 C_A\left(\frac{-1}{t_i t_{i+1}}V^\mu(q_i,q_{i+1})V_\mu(q_i,q_{i+1}) - \frac{4}{\mathbf{p}_i^2}\,\theta\left(\mathbf{p}_i^2<\lambda^2\right)\right)\right) \cdot \prod_{j=1}^{n-1}\exp\left[\omega^0(q_j,\lambda)(y_j-y_{j+1})\right]$$

Skeleton/Born Process

Resolved Real Emissions

Virtual + Unresolved Real (finite)

$$\sigma_{2j}^{\rm resum,match} = \sum_{f_1,f_2}\ \sum_{n=2}^{\infty}\ \prod_{i=1}^{n}\left(\int_{p_{i\perp}=\lambda}^{p_{i\perp}=\infty}\frac{\mathrm{d}^2\mathbf{p}_{i\perp}}{(2\pi)^3}\ \int\frac{\mathrm{d}y_i}{2}\right)\ \frac{\overline{\left|\mathcal{M}_{\rm HEJ}^{f_1 f_2\to f_1 g\cdots g f_2}(\{p_i\})\right|}^2}{\hat{s}^2} \times \sum_m \mathcal{O}^e_{mj}(\{p_i\})\ w_{m-\rm jet} \times\ x_a f_{A,f_1}(x_a,Q_a)\ x_2 f_{B,f_2}(x_b,Q_b)\ (2\pi)^4\ \delta^2\left(\sum_{i=1}^{n}\mathbf{p}_{i\perp}\right)\ \mathcal{O}_{2j}(\{p_i\})$$

Merging

$$\sigma_{2j} = \sigma_{2j}^{\rm resum,match} + \sum_n \sigma_{nj}^{\rm non-FKL}$$

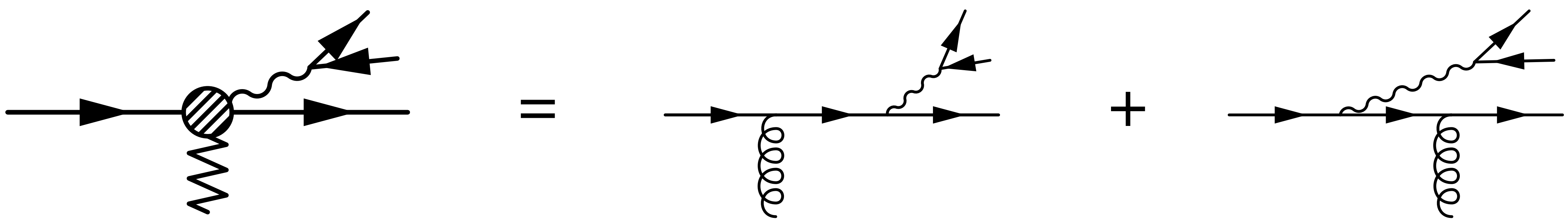
Matching

Andersen & JMS arXiv:0908.2786, 0910.5113, 1101.5394

Beyond pure jets

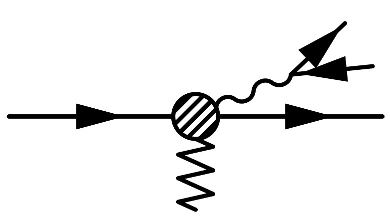
The method extends to other X+jets processes:

E.g. W+jets

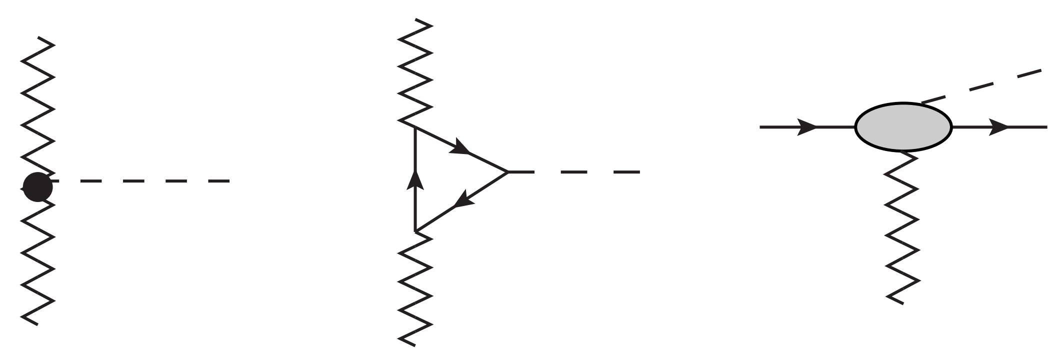


Andersen, Hapola & JMS arXiv:1206.6763

Also



Z+jets



H+jets

Andersen, Medley & JMS arXiv:1603.05460

Andersen, Cockburn, Heil, Maier & JMS arXiv:1812.08072

HEJ Principles

The HEJ description is:

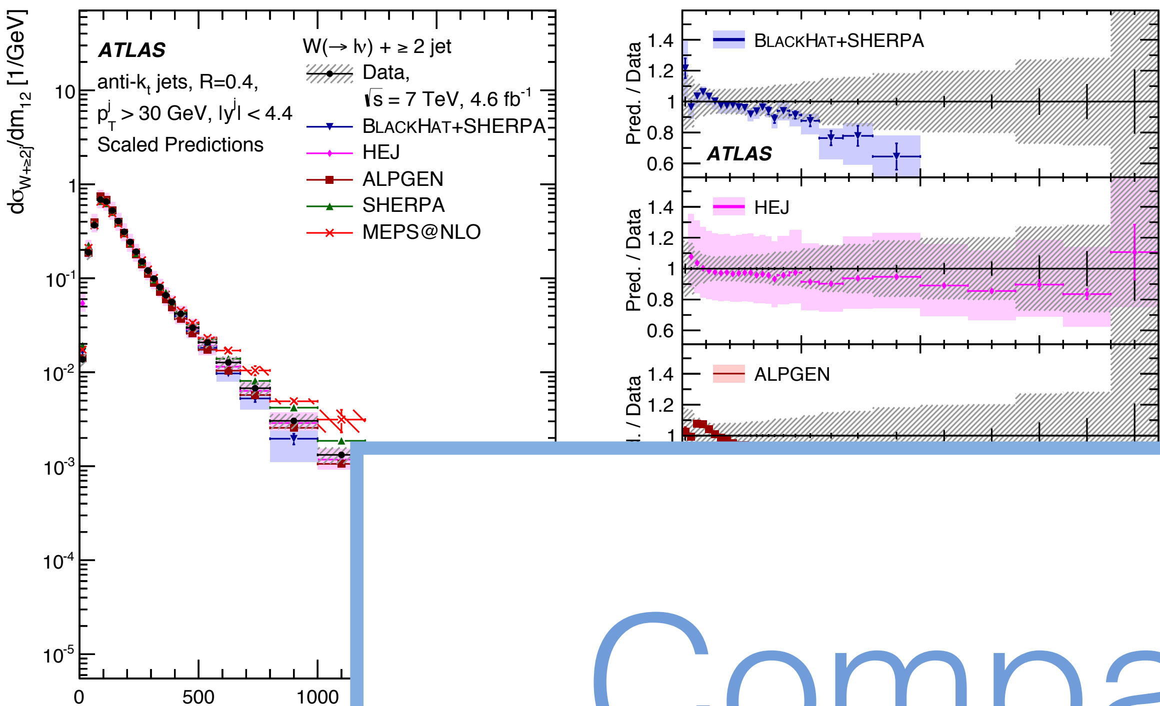
- exact for simple processes (2 to 2 (+X))
- gauge invariant in all phase space
- sufficiently fast for numerical integration (up to 30 gluons)
- accurate to leading logarithm in s/t
- merged with LO samples (2j, 3j, 4j, … taken in LHE format)

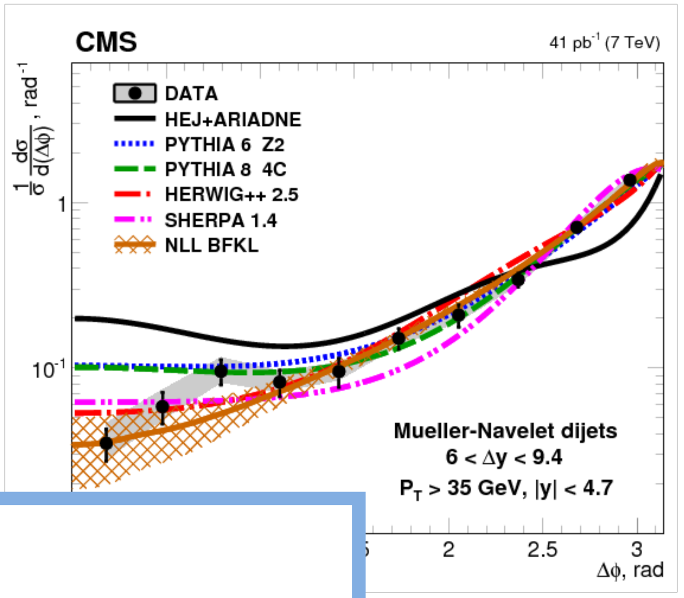
HEJ2: fully flexible (exclusive) MC event generator for
jets & H+dijets
compatible with LHAPDF, Rivet, fastjet, …

http://hej.web.cern.ch

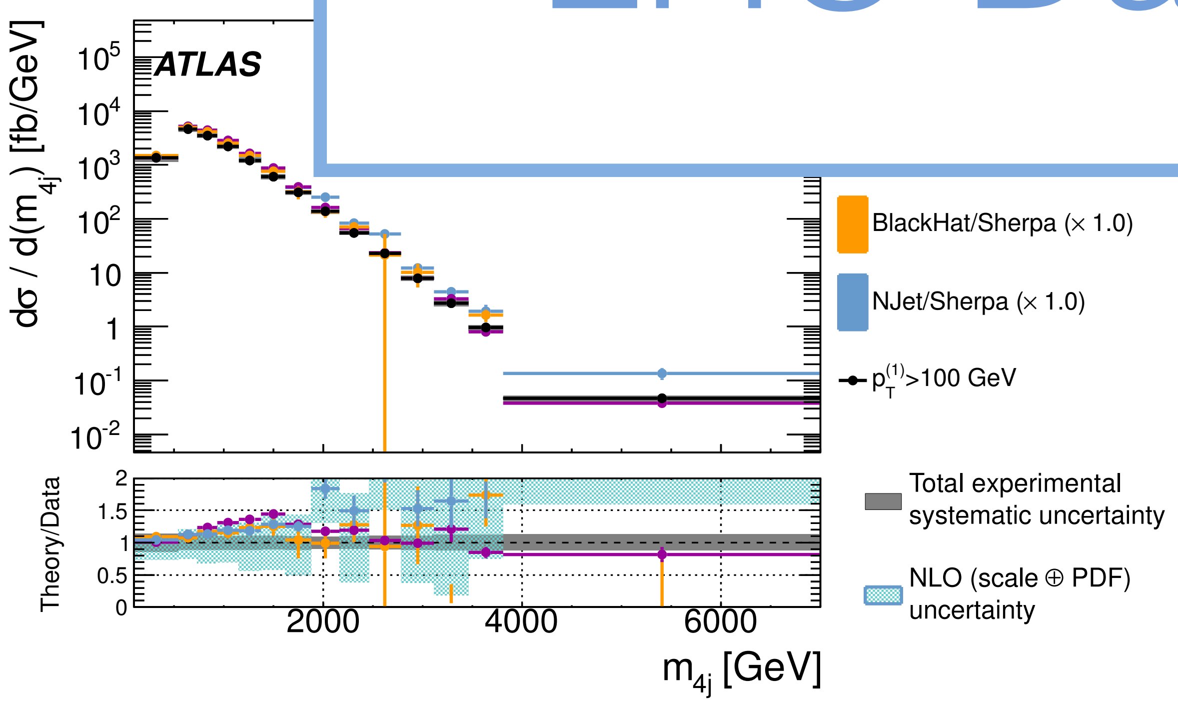
W+dijets and Z+dijets available in HEJ1, soon in HEJ2

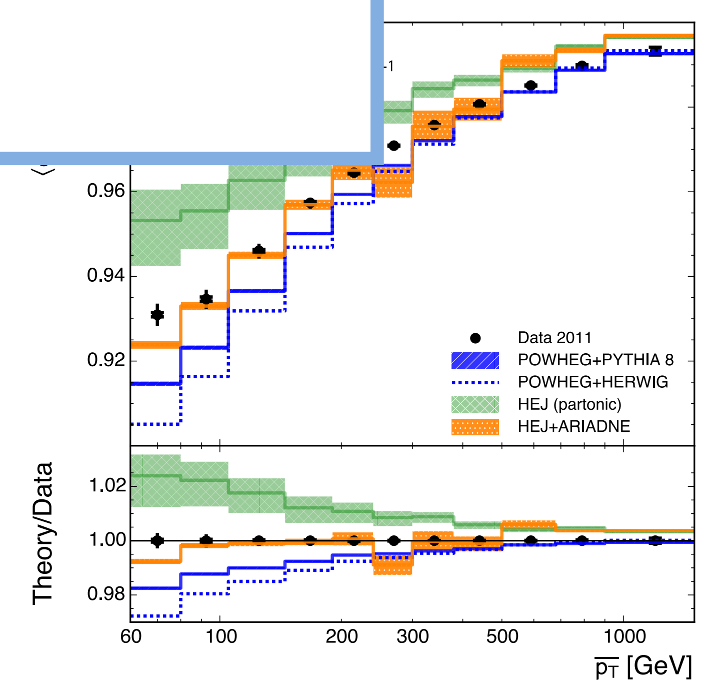
Andersen, Hapola, Heil, Maier & JMS arXiv:1902.08430



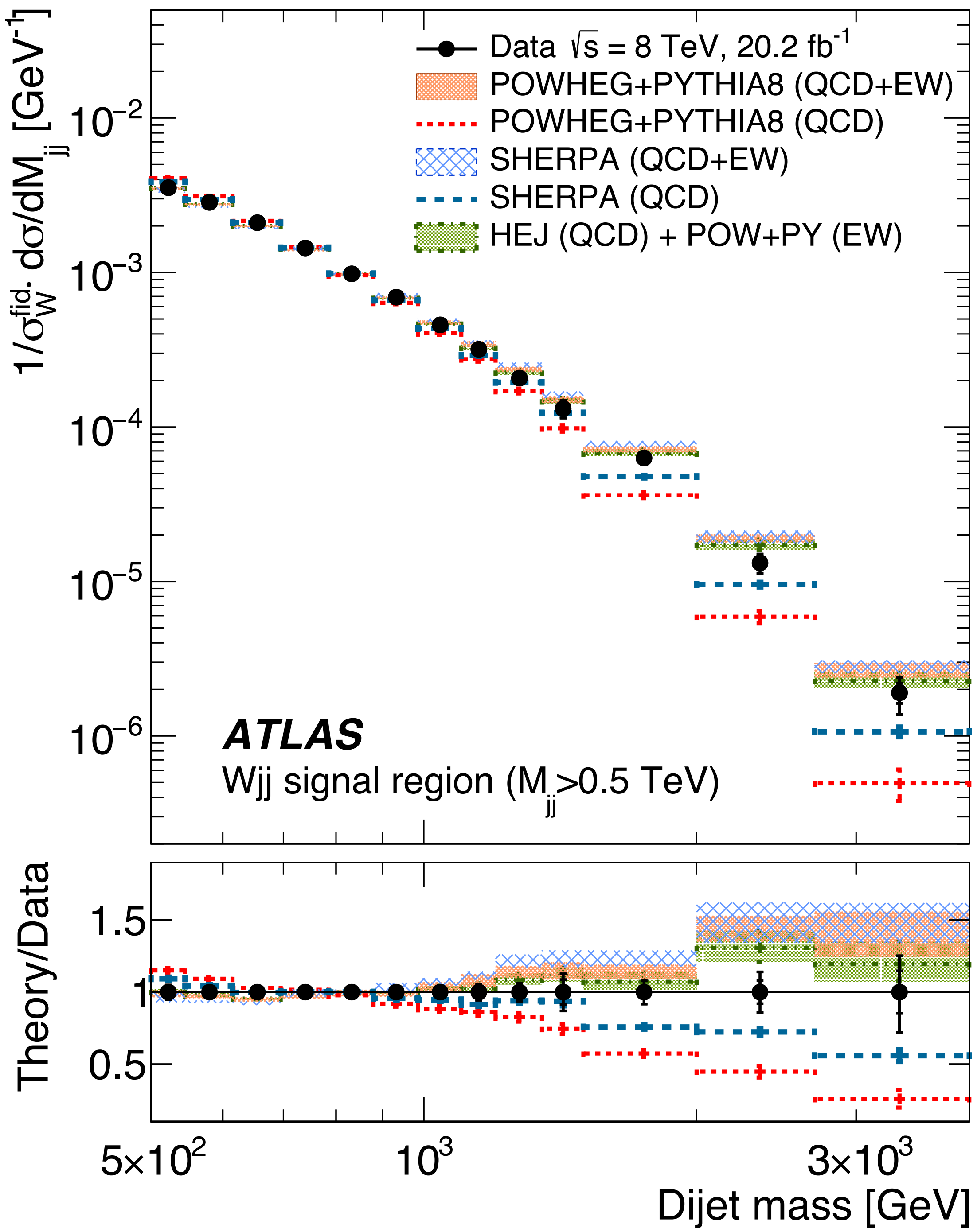


Comparisons to LHC Data so far





2017 ATLAS W+2j

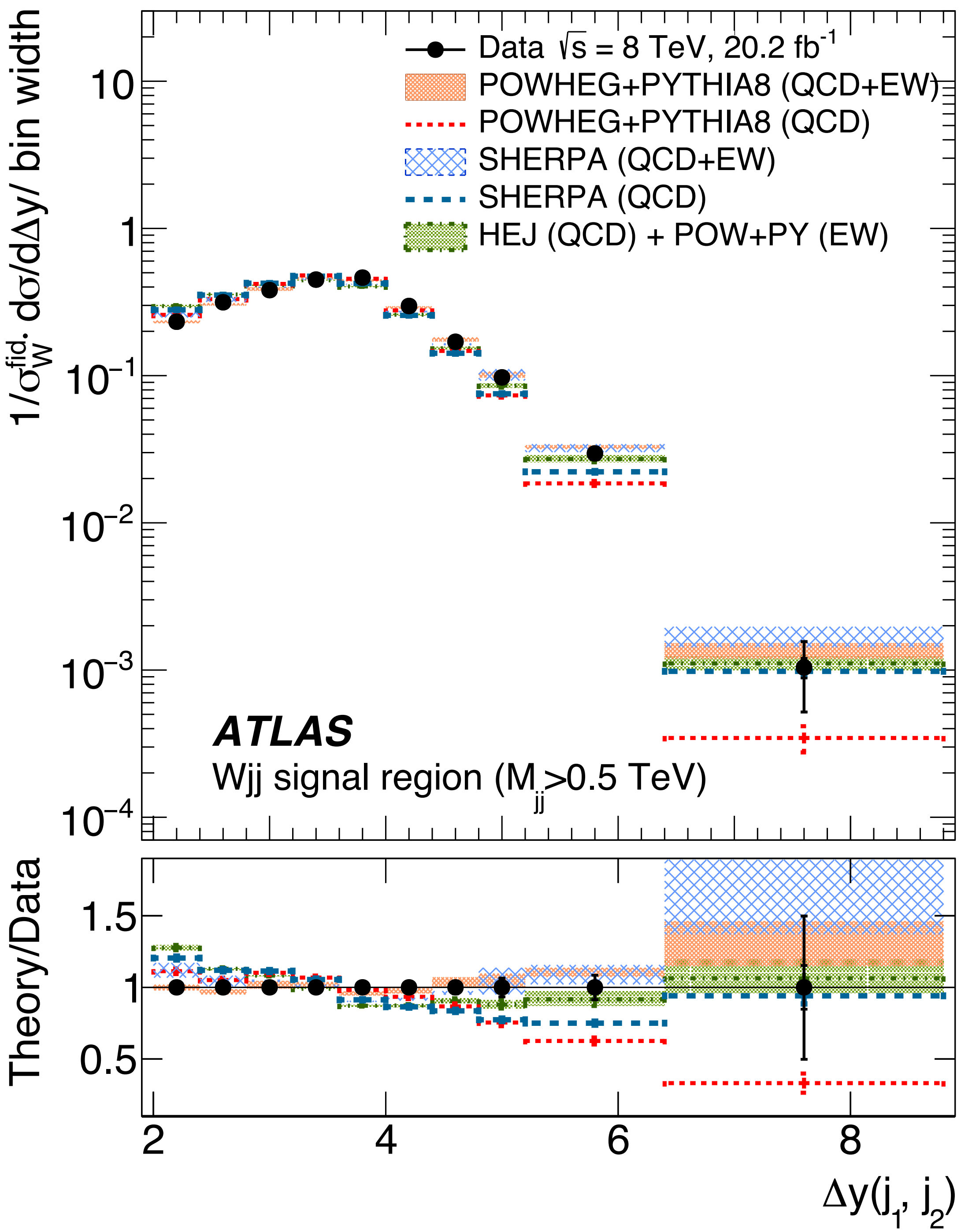


ATLAS arXiv:1703.04362

W+2j study to investigate separation of QCD/EW contributions compared to NLO+PS (Powheg/Sherpa) and HEJ+EW from Powheg

- QCD contribution decreases at large dijet mass, but remains significant
- NLO+PS slightly overshoot, and increasing

2017 ATLAS W+2j



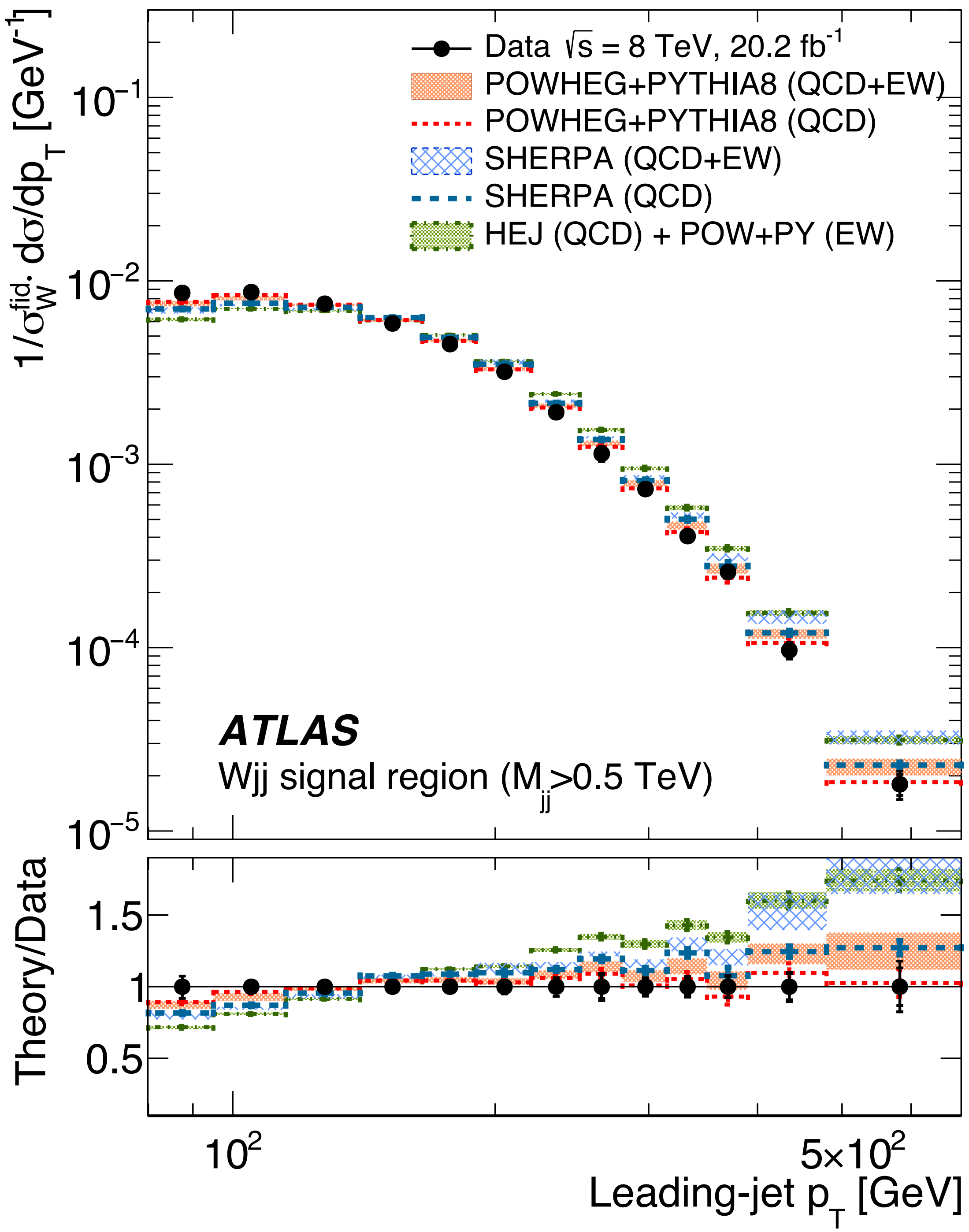
ATLAS arXiv:1703.04362

- Similar conclusions when plotted as a function of rapidity separation of hardest jets
- Similarity between Powheg and HEJ also seen in earlier jet studies, despite very different construction

for discussion see Andersen et al arXiv:1202.1475

2017 ATLAS W+2j

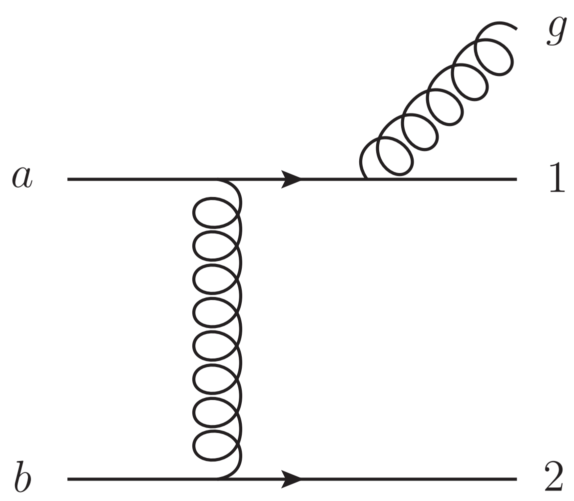
- QCD contribution is no longer suppressed compared to EW
- No systematic evolution in p_T in HEJ, and in regions of large p_T the description is poorer
- Adding formerly subleading contributions to HEJ will help here



ATLAS arXiv:1703.04362

HEJ Beyond LL

- Have seen HEJ description worsens in regions where matching component is more significant, e.g. large momentum
- Description already leading-log in inclusive (X+)dijets, but is <u>not</u> leading-log for all subprocesses
- Adding these will move more of the cross section into part subject to resummation



$+ \ldots$

Example: allow a gluon emission outside in rapidity of a quark

Which "all-order"?

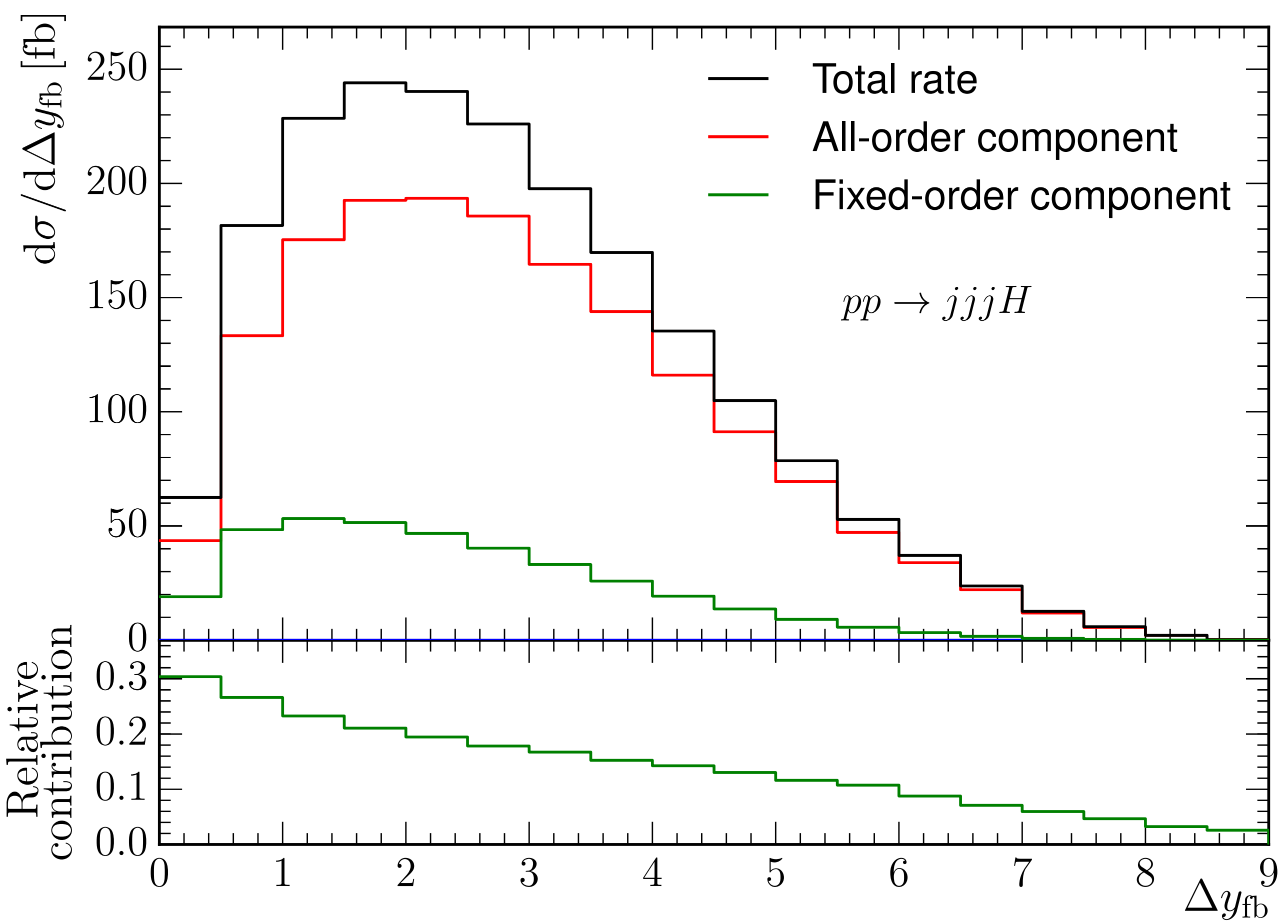
$$|M_{2j}|^2 = \alpha_s^2 \left(a_2(\hat{s}^2/\hat{t}^2) + b_2\right)$$
$$+ \alpha_s^3 \left(a_3(\hat{s}^2/\hat{t}^2) \log(\hat{s}/\hat{t}) + b_3(\hat{s}^2/\hat{t}^2) + c_3\right)$$
$$+ \alpha_s^4 \left(a_4(\hat{s}^2/\hat{t}^2) \log^2(\hat{s}/\hat{t}) + b_4(\hat{s}^2/\hat{t}^2) \log(\hat{s}/\hat{t}) + \ldots\right)$$
$$+ \ldots$$

We will obtain some of these

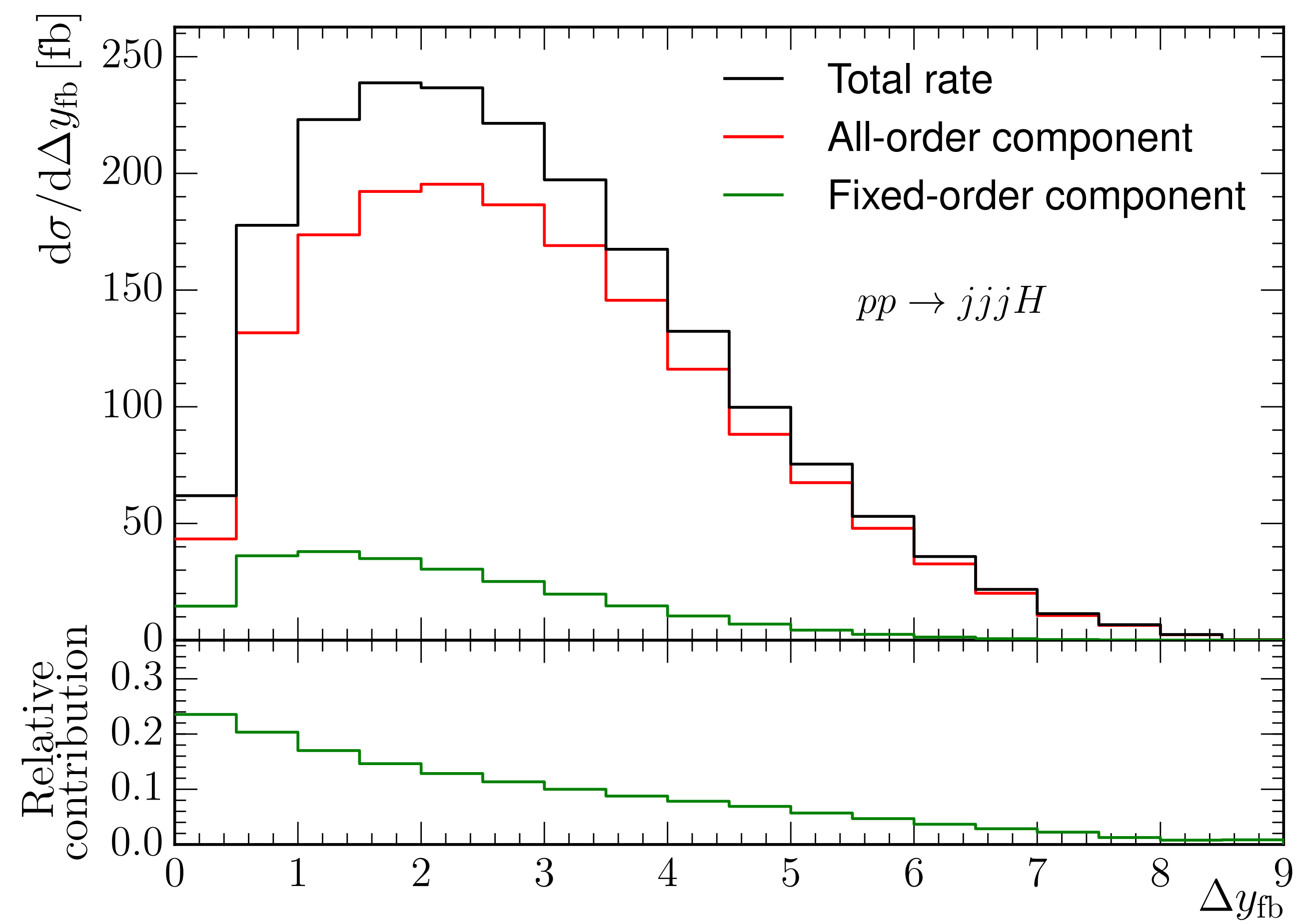
- LO = first line
- NLO = first two lines
- Leading logs = the 'a'-terms
- Our description = **LO + LL** (plus NLO cross section, for now…)

Impact in Higgs+jets

Without unordered resummation



With unordered resummation



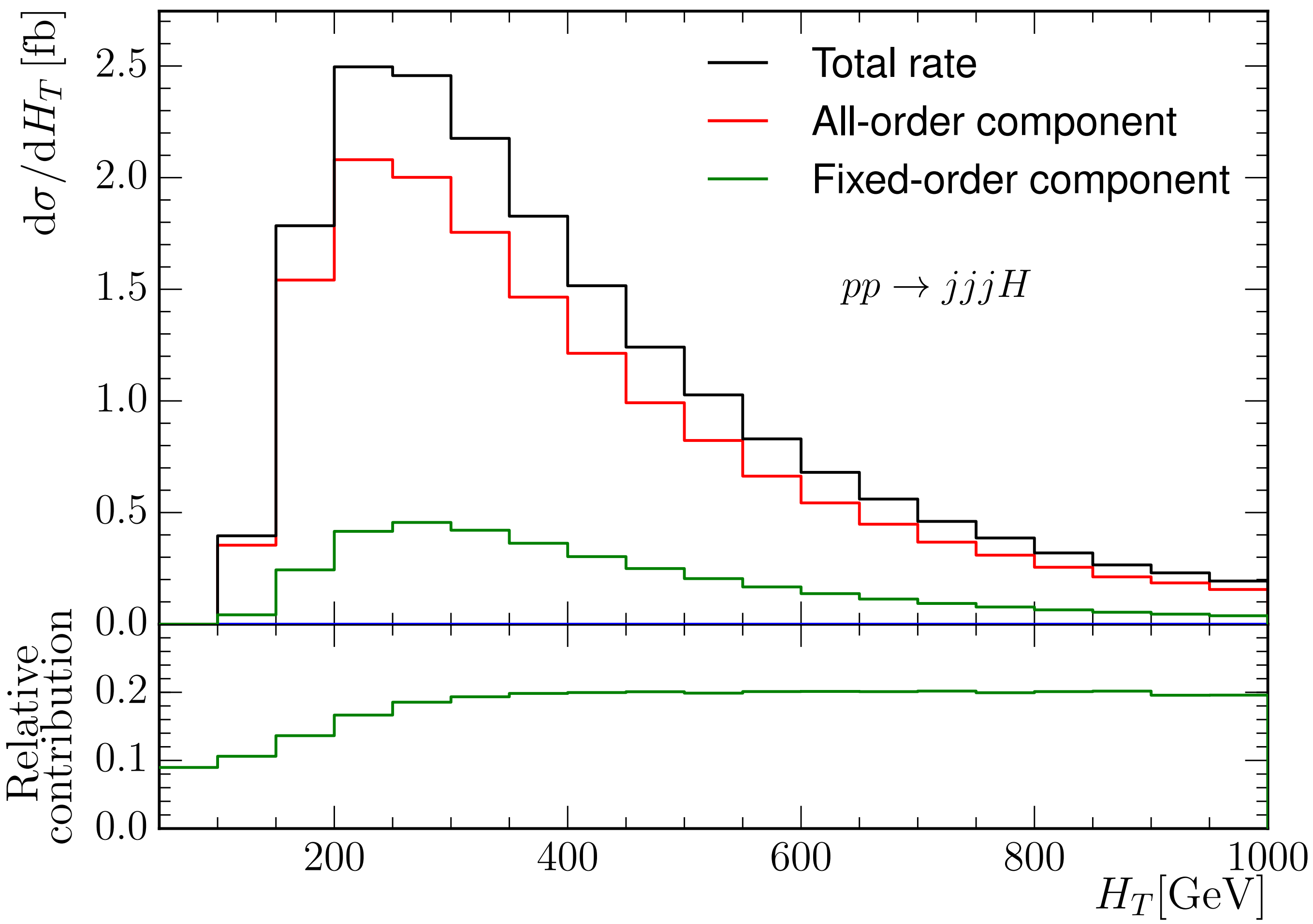
Drop with Δy_{fb} illustrates dominant LL in HE limit in all-order part

With un-ordered correction, fixed-order starts lower and drops faster

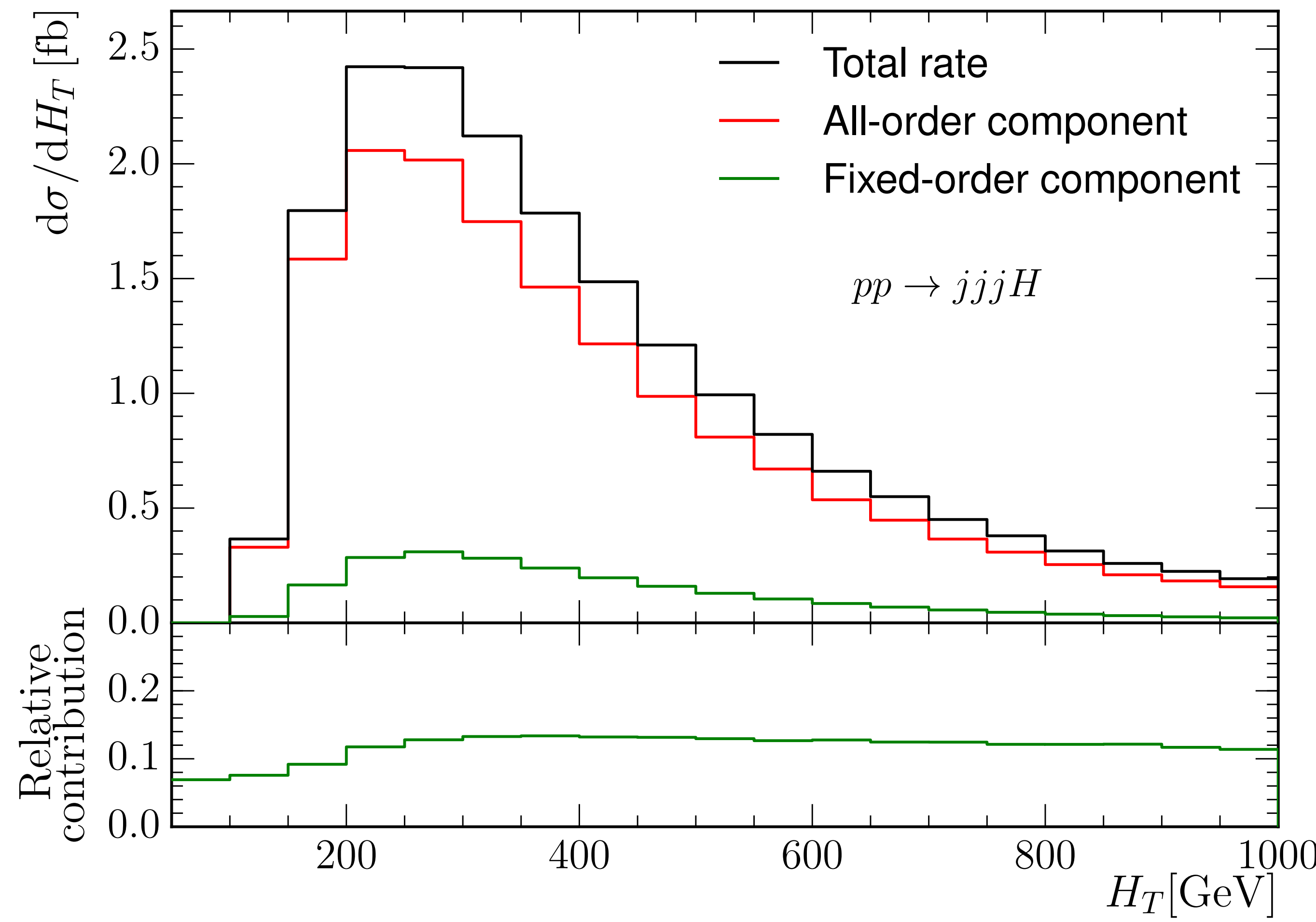
Andersen, Hapola, Maier, JMS arXiv:1706.01002

Impact in Higgs+jets

Without unordered resummation



With unordered resummation

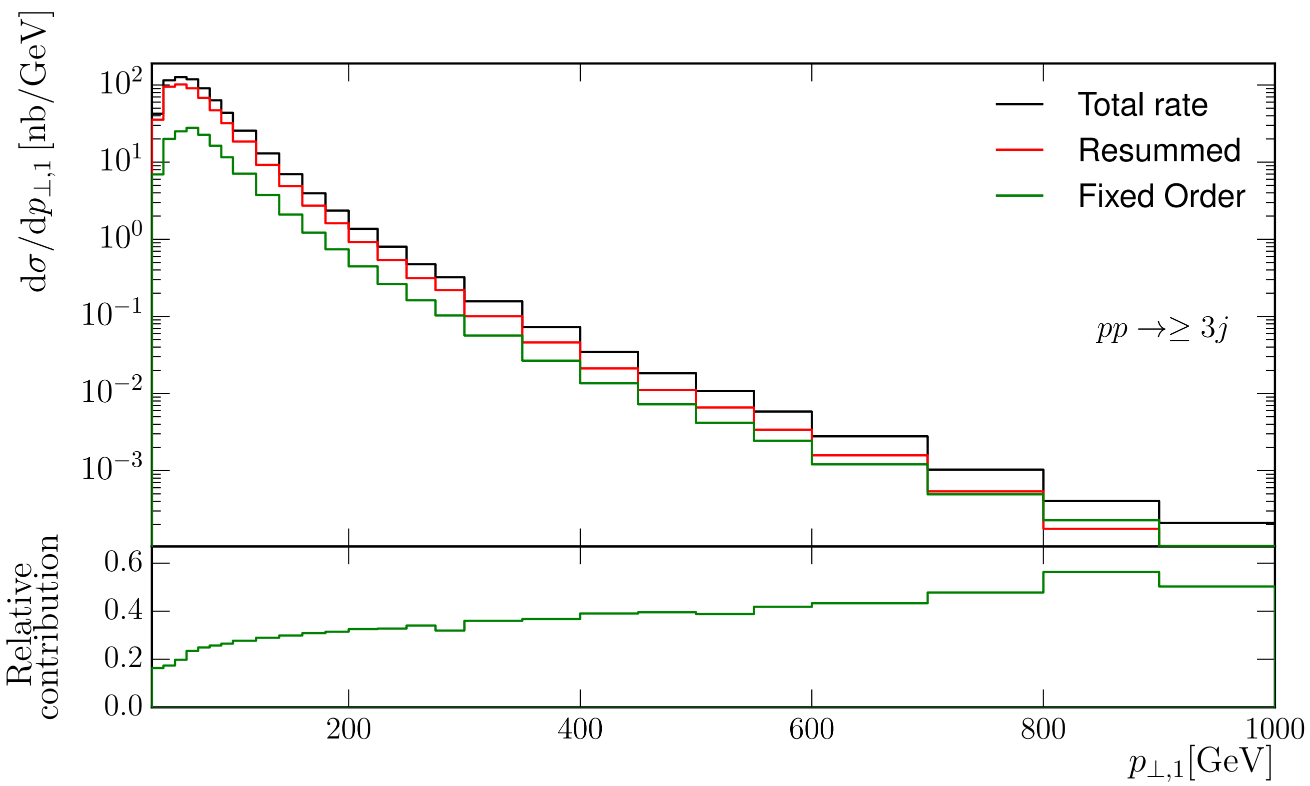


Fixed order component reduced from ~20% to ~12%

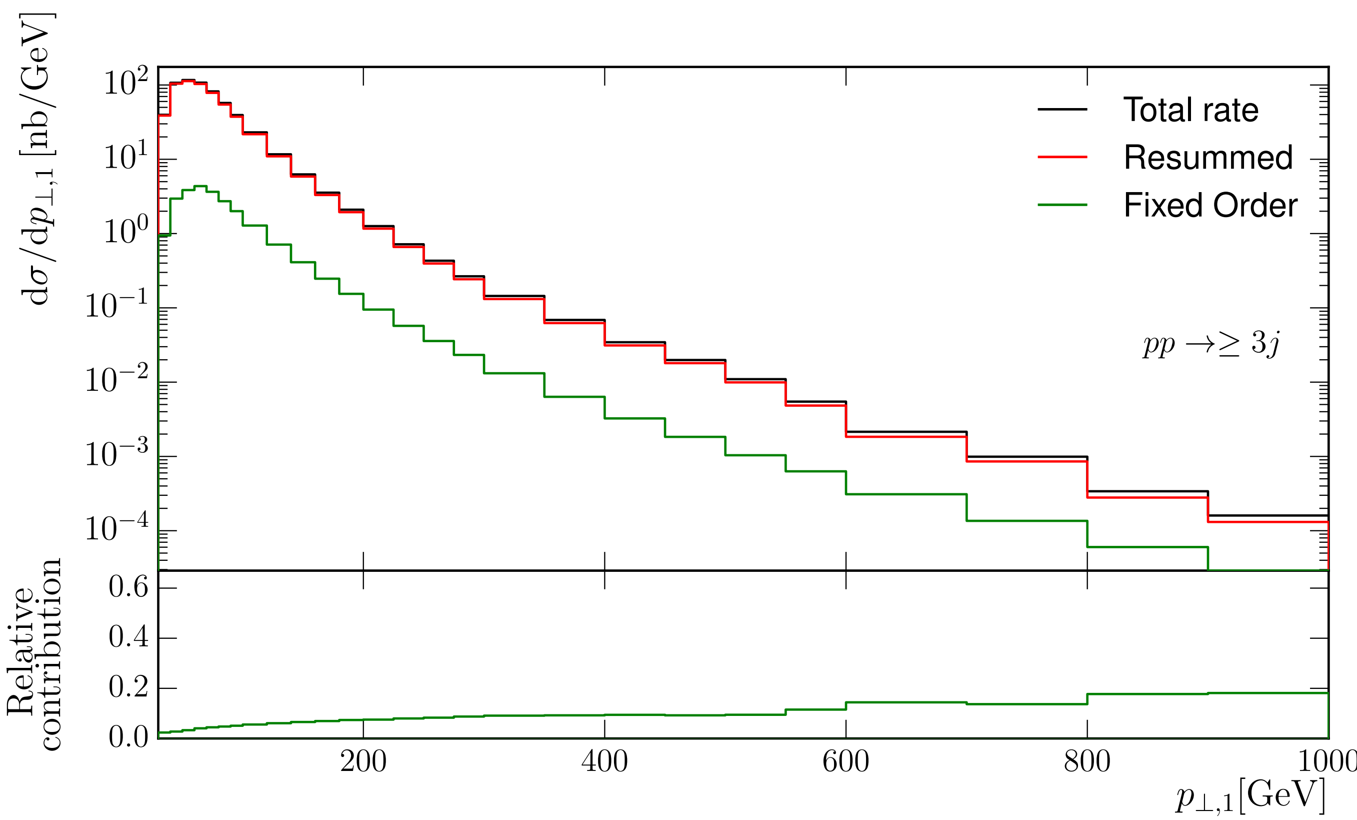
Andersen, Hapola, Maier, JMS arXiv:1706.01002

Impact in Incl 3-Jets

Without NLL resummation



With NLL resummation



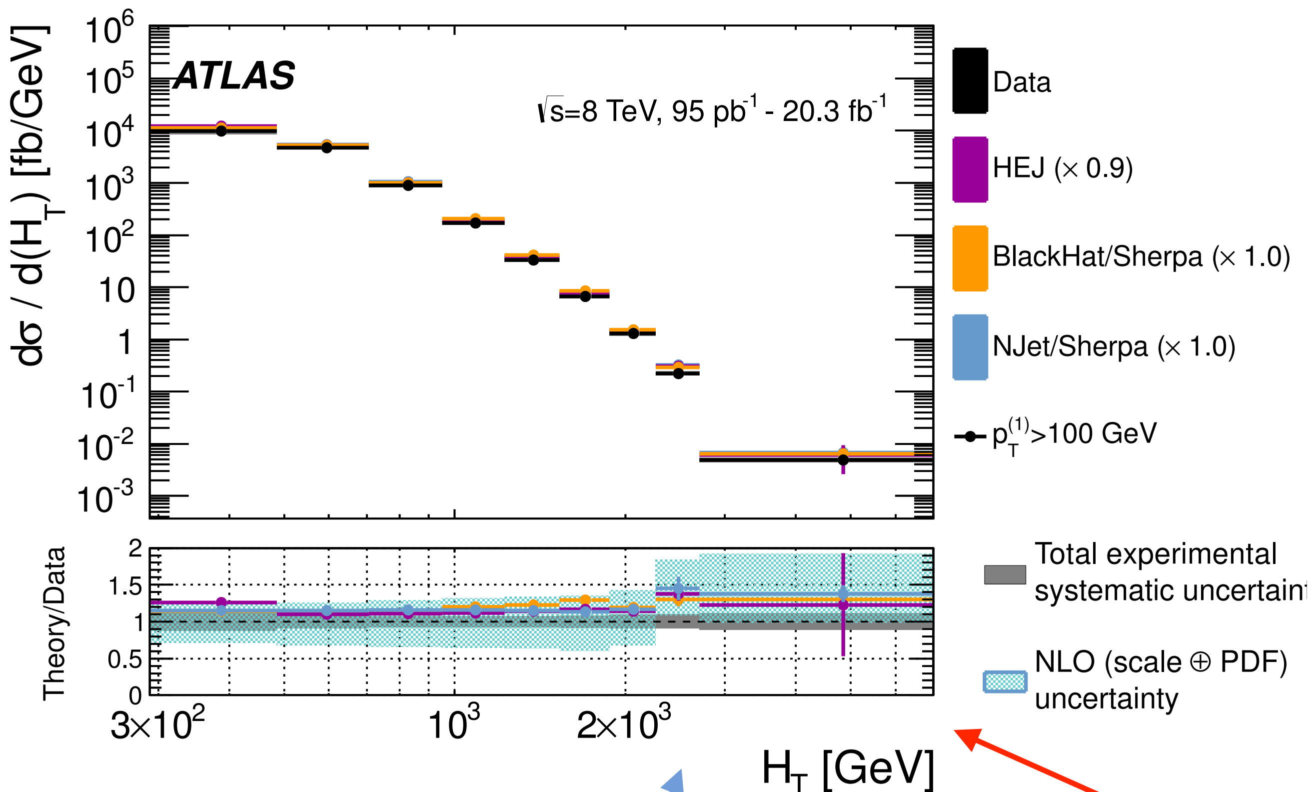
Max fixed order component reduced here from ~50% to ~20%

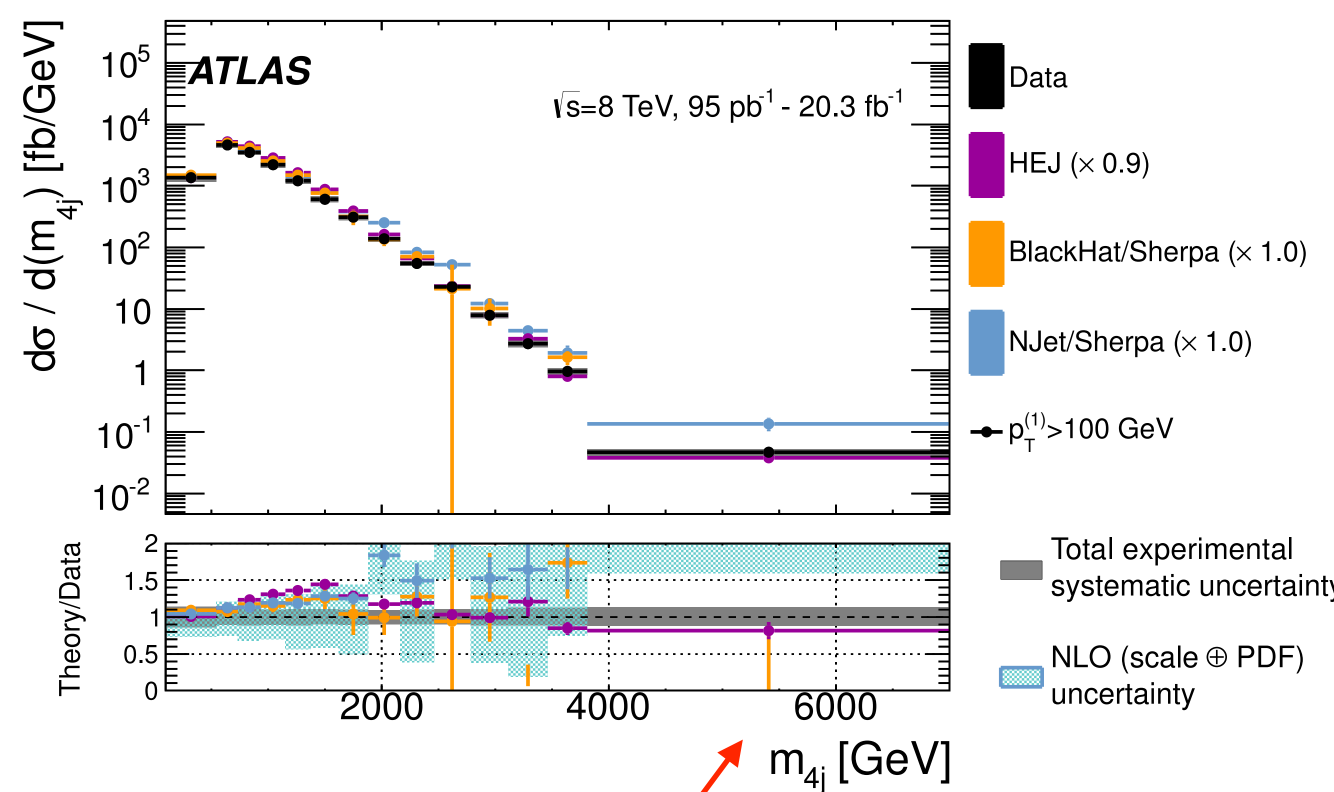
J.Cockburn: PhD Thesis

ATLAS 4-Jet Analysis - 8 TeV

The first exp. analysis where HEJ predictions include this correction

ATLAS arXiv:1509.07335

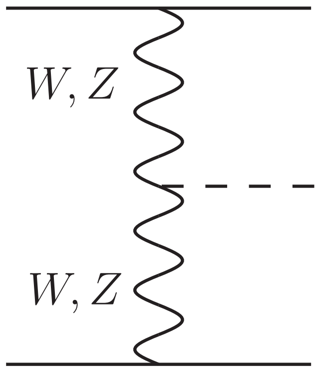


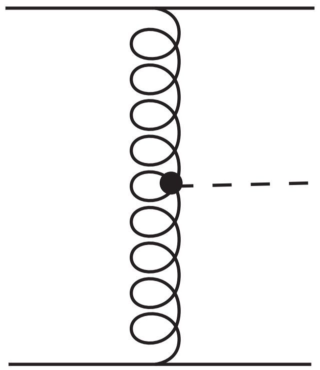


Large momentum usually difficult region for HEJ: good description here

Very high energies measured (already at Run I)

Higgs Boson + Dijets

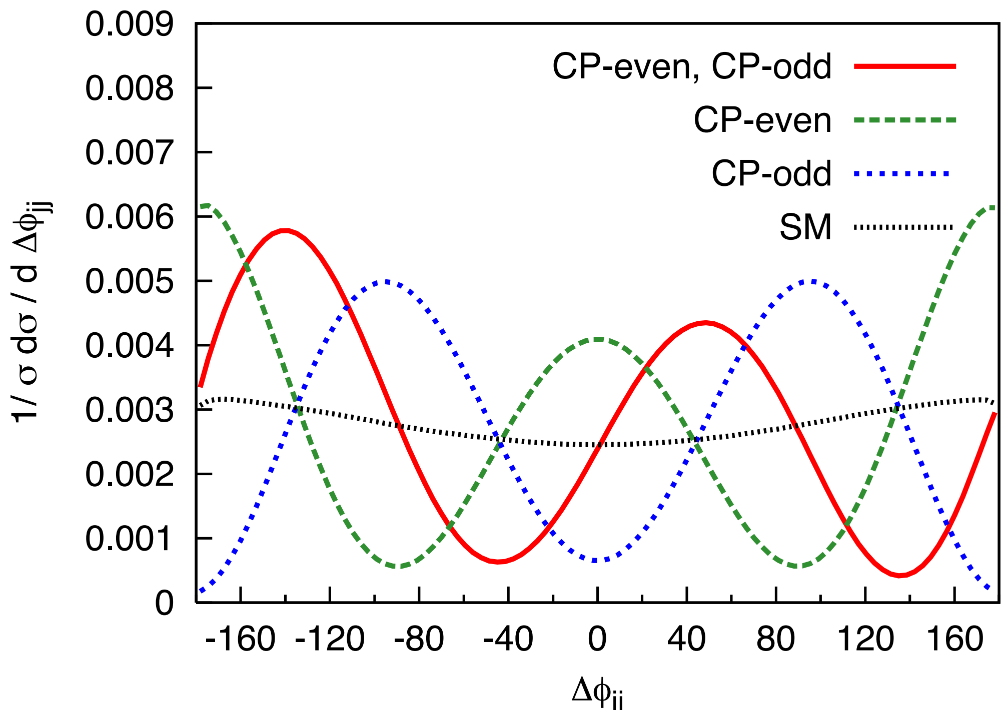




Higgs boson looks like SM so far, but critical to check CP structure of couplings to bosons

Azimuthal angle between the dijets is sensitive to this

Figy et al hep-ph/0609075



Use distinctive event shape to separate channels with "VBF cuts"

e.g. $\Delta y_{jj} > 2.8, \ m_{jj} > 400 \text{ GeV}$

BUT this precisely enhances higher orders in pert. expansion

Finite Quark Masses in H+2j

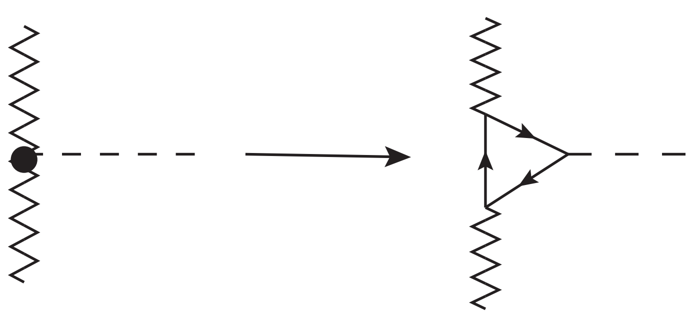
Fixed-order stalled for full quark mass effects because LO = 1-loop.

LO results only for 2 and 3 jets (no NLO)

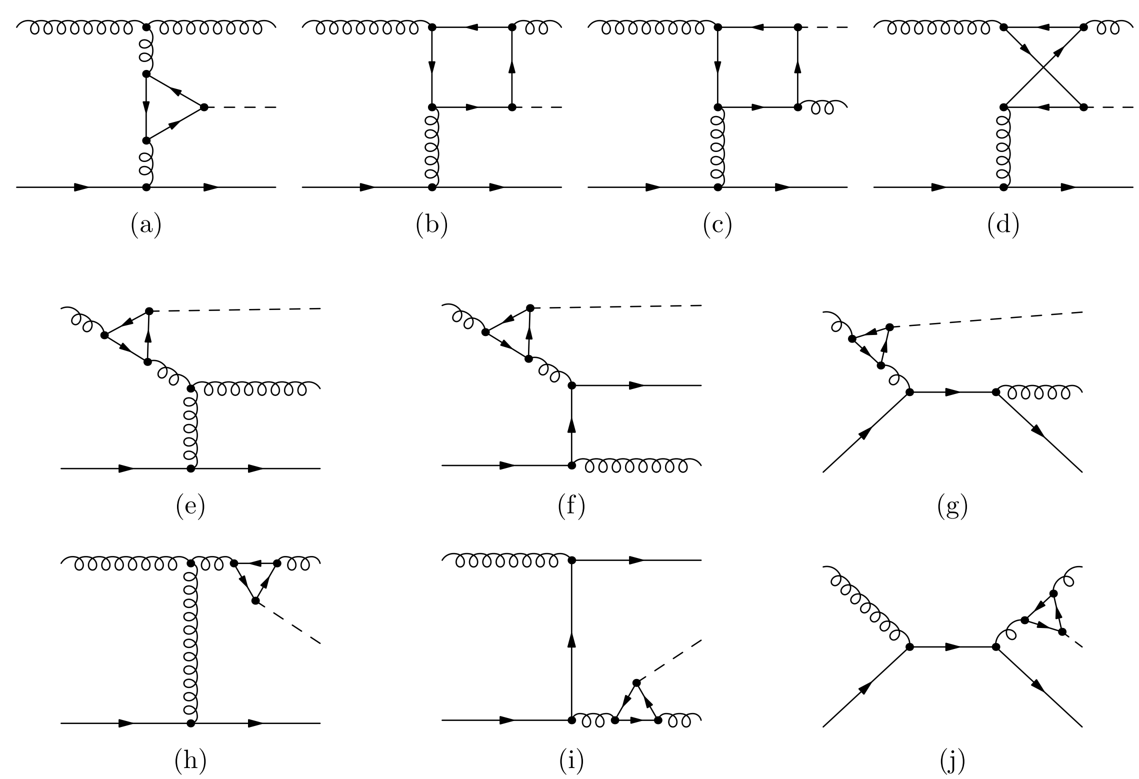
Del Duca et al hep-ph/0105129, hep-ph/0108030
Greiner et al arXiv:1608.01195

In HEJ, factorised structure removes complexity from increasing number of jets

Andersen, Cockburn, Heil, Maier & JMS arXiv:1812.08072



Straight-forward
e.g. $qQ \to qHQ$



External Higgs more involved but calculated

Finite Quark Masses in HEJ

Performed at amplitude level so we include mass effects from top quark, bottom quark *and* the interference between the two

Fixed-order matching performed to highest-available accuracy

Here use Sherpa and OpenLoops

Gleisberg et al arXiv:0811.4622; Cascioli, Maierhöfer, Pozzorini arXiv:1111.5206

Highest available =

finite m_t $H + 2j$ at LO   ($3j$ results exist, but not usable)

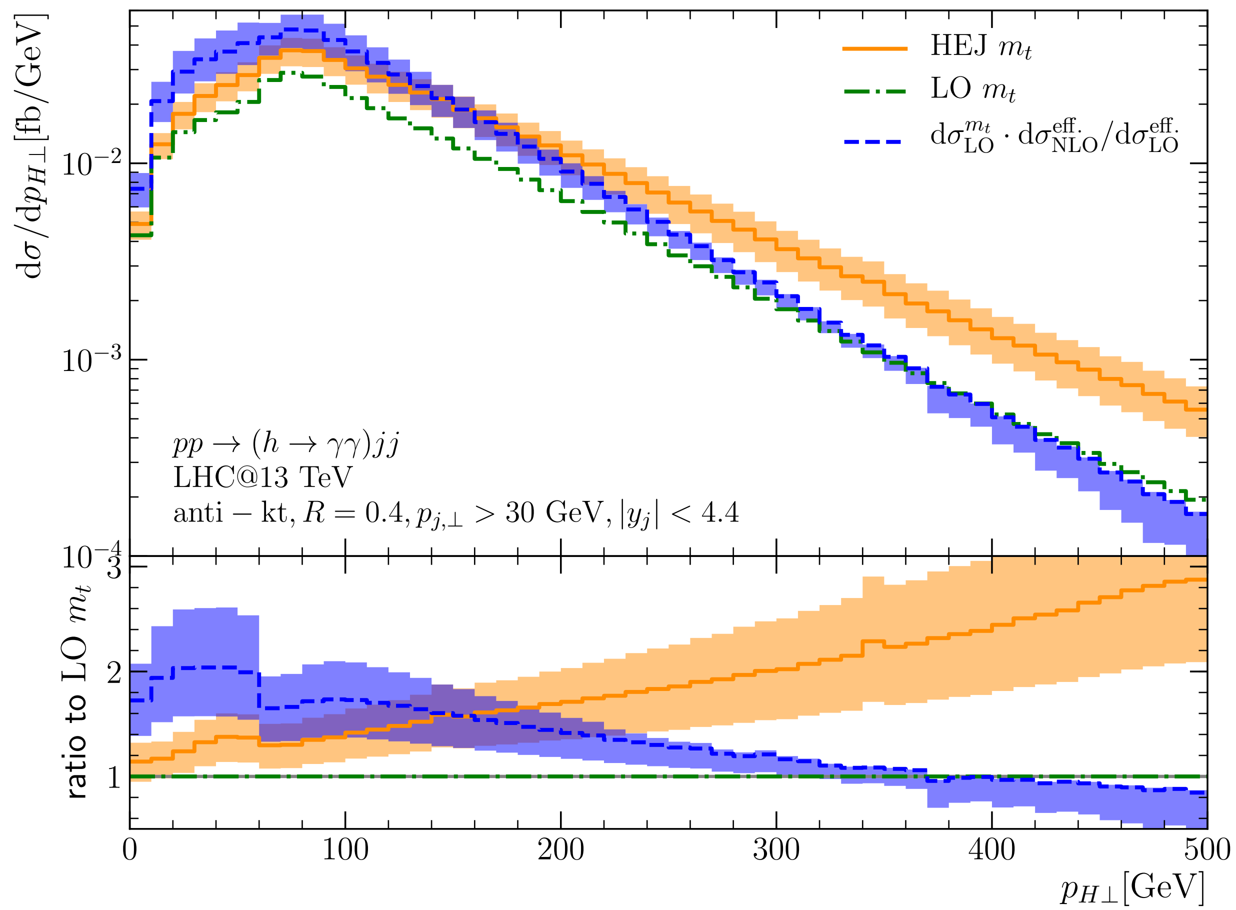
infinite m_Q $H + 2j$ at NLO

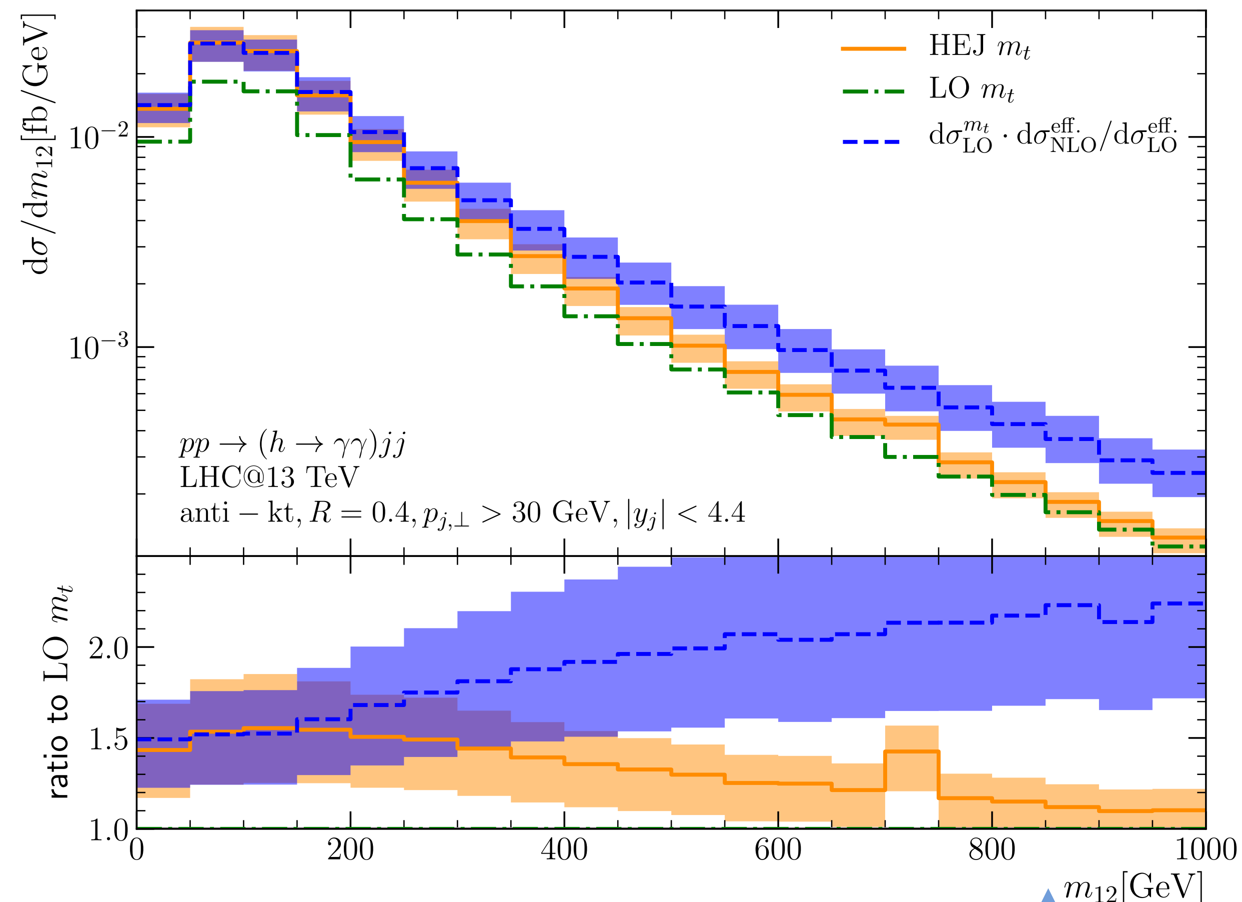
$H + 5j$ at LO

Finite Quark Mass Results

First probe the impact of higher orders in α_s **Andersen, Cockburn, Heil, Maier & JMS arXiv:1812.08072**

HEJ here temporarily without m_b





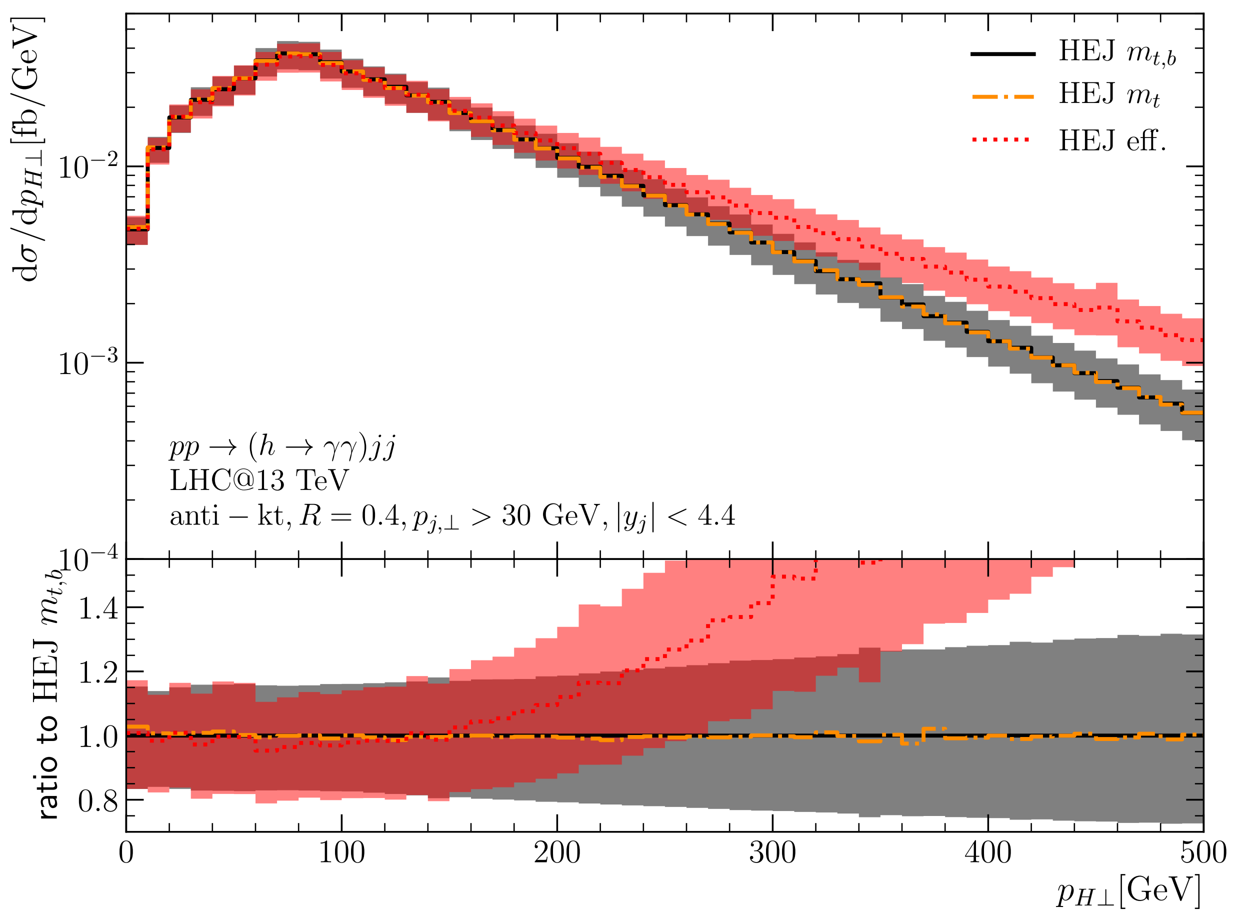
NLO K-factors clearly not flat, HEJ harder $p_{H\perp}$ spectrum

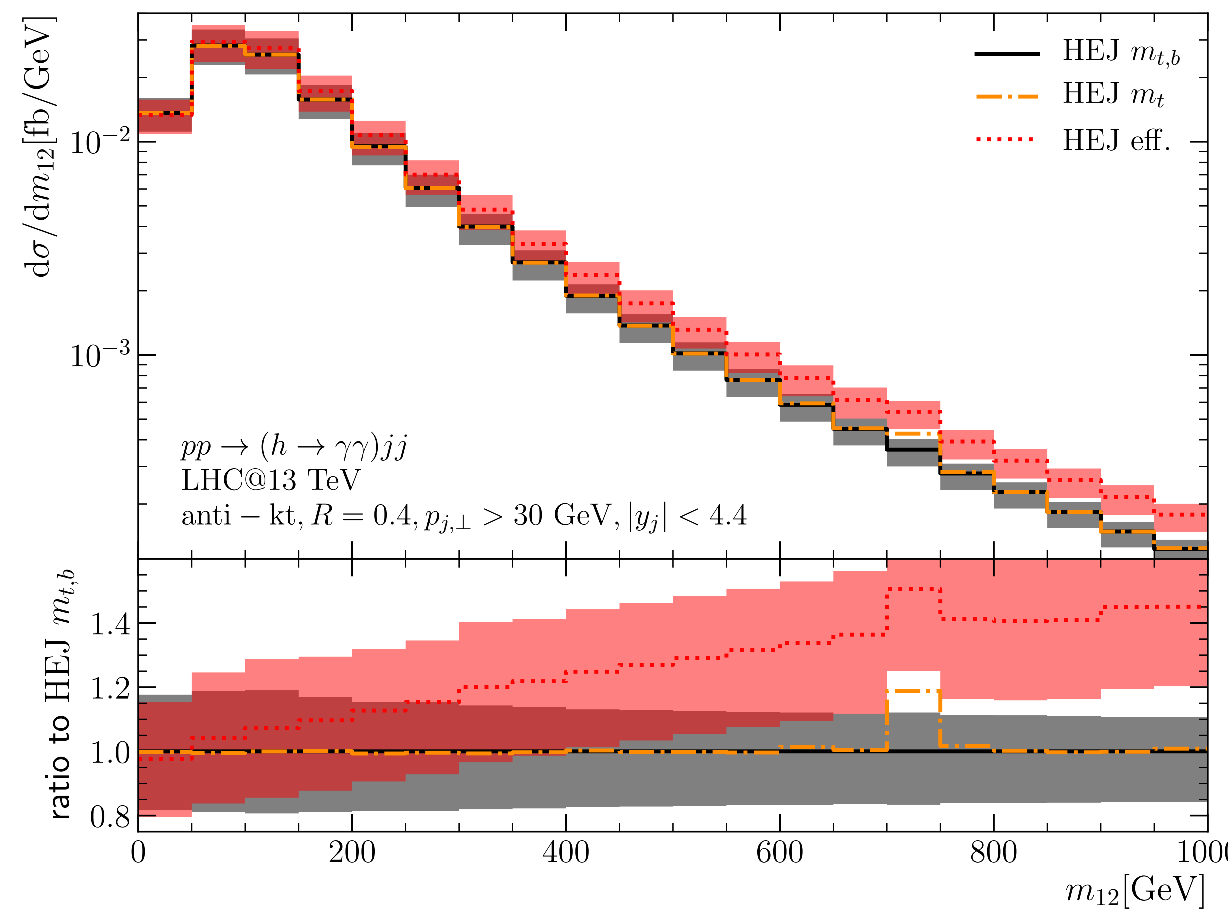
HEJ much steeper drop with m_{12}, NLO missing log-suppression of 3j part

Finite Quark Mass Results

Now probe the impact of quark masses

Andersen, Cockburn, Heil, Maier & JMS arXiv:1812.08072





Importance of finite quark mass increases with $p_{H\perp}$

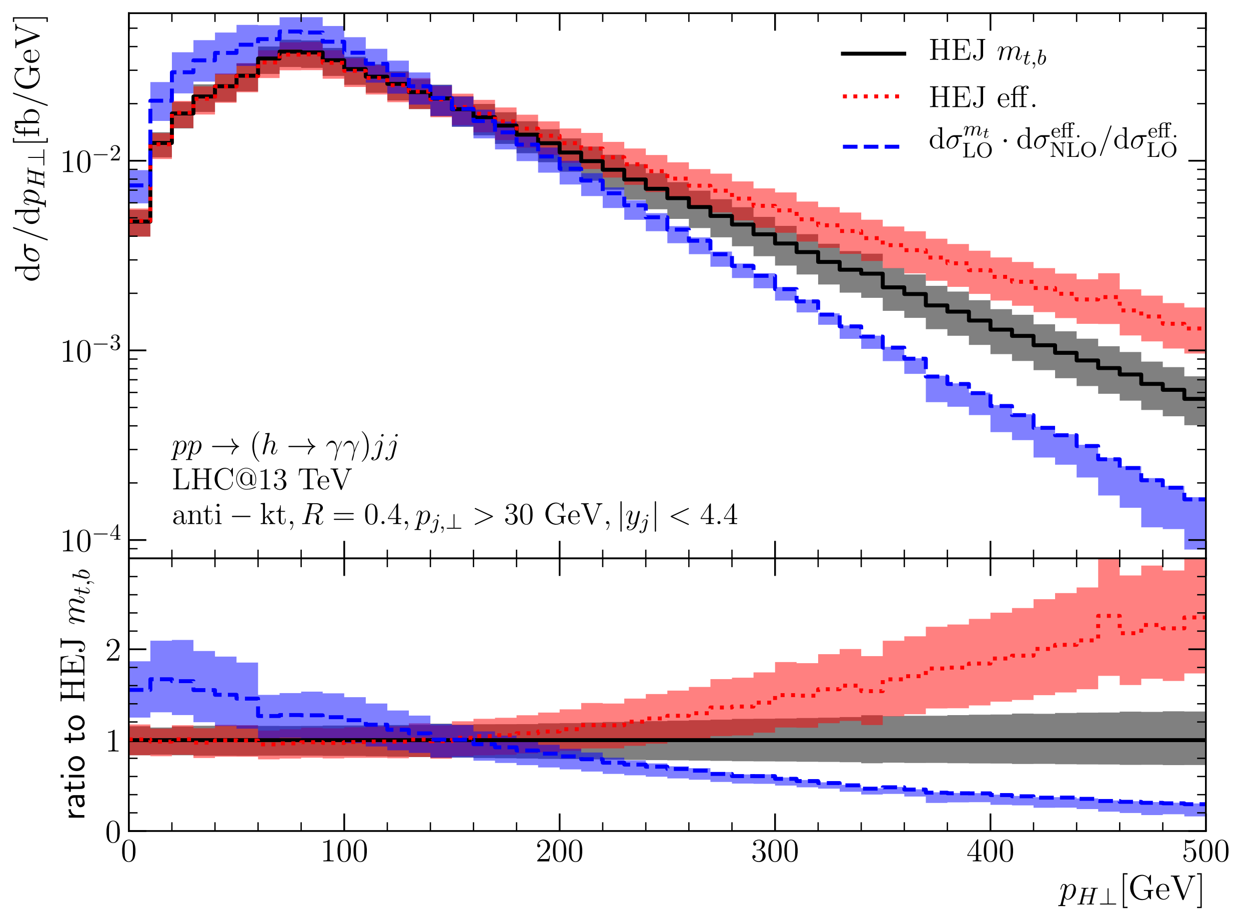
Relatively small impact of m_b, finite m_t lowers predictions at large m_{12}

Therefore finite quark mass effects make VBF cuts more effective

Finite Quark Mass Results

Full HEJ prediction vs “best” fixed-order

Andersen, Cockburn, Heil, Maier & JMS arXiv:1812.08072



Finite quark mass effects - significant reduction (up to 50%)

HEJ $p_{H\perp}$ spectrum much harder - increases sensitivity to loop momenta

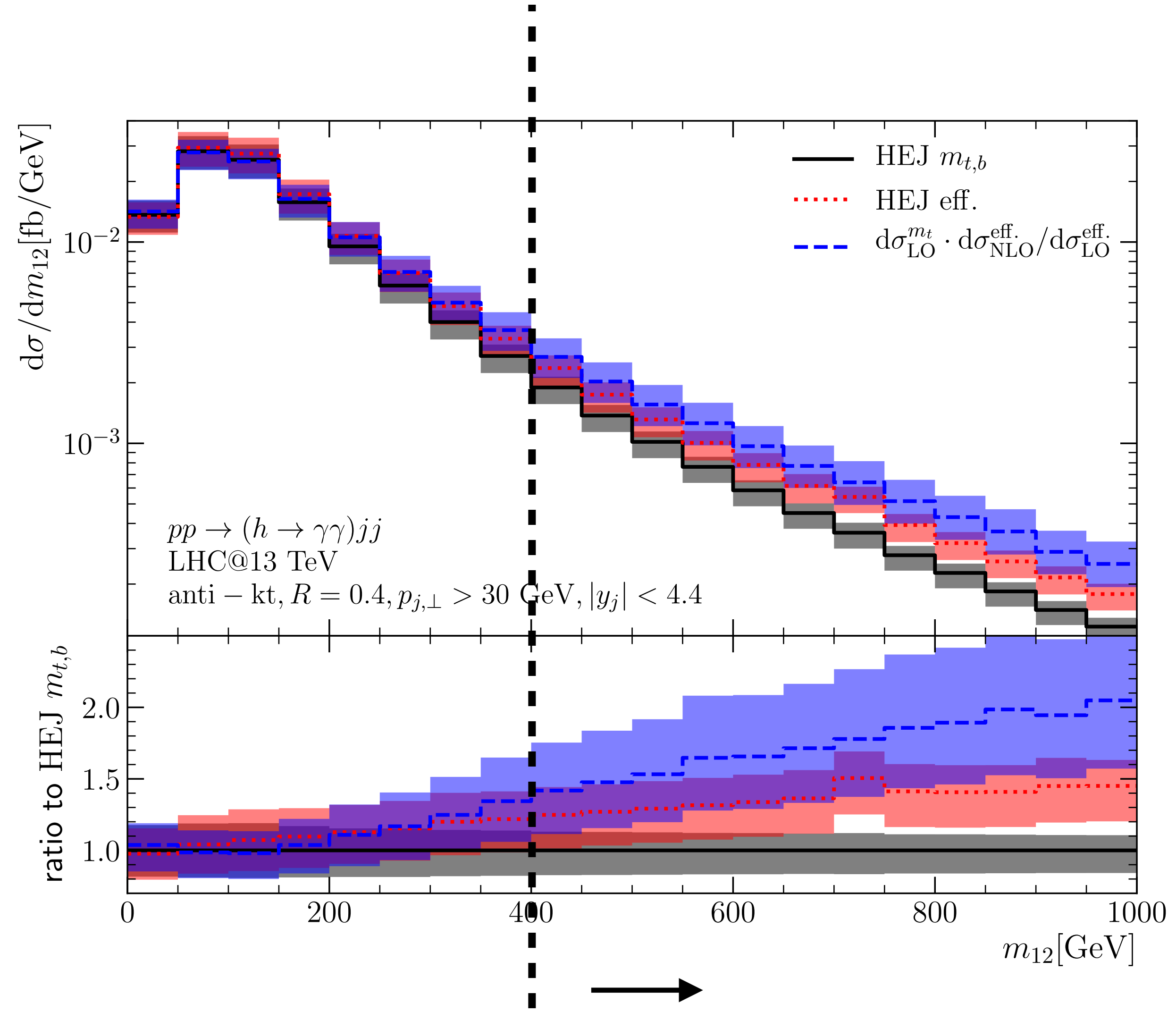
Finite Quark Mass Results

Full HEJ prediction vs "best" fixed-order

Andersen, Cockburn, Heil, Maier & JMS arXiv:1812.08072

- Resummation alone reduces cross section at large values
- Also gives harder $p_{H\perp}$ spectrum which enhances finite quark mass effects which reduce x-section in VBF cuts by *further* 11%

Prediction	xs after VBF cuts
Fixed order	9%
HEJ	4%



Typical VBF cut

Conclusions

- Huge phase space for extra hard jets, and for enhancements of higher-order coefficients which damage convergence of fixed-order expansion
- The effect is visible in LHC Data (also TeVatron)
- Sizable effects and implications for Higgs VBF analyses
- HEJ allows inclusion of finite quark mass effects combined with all order predictions:
  Find VBF cuts more severe than fixed-order estimates
- Public code, documentation, sample analyses, …

https://hej.web.cern.ch