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Automatically Building Better Algorithms :

taking existing computer programs and automatically improving them

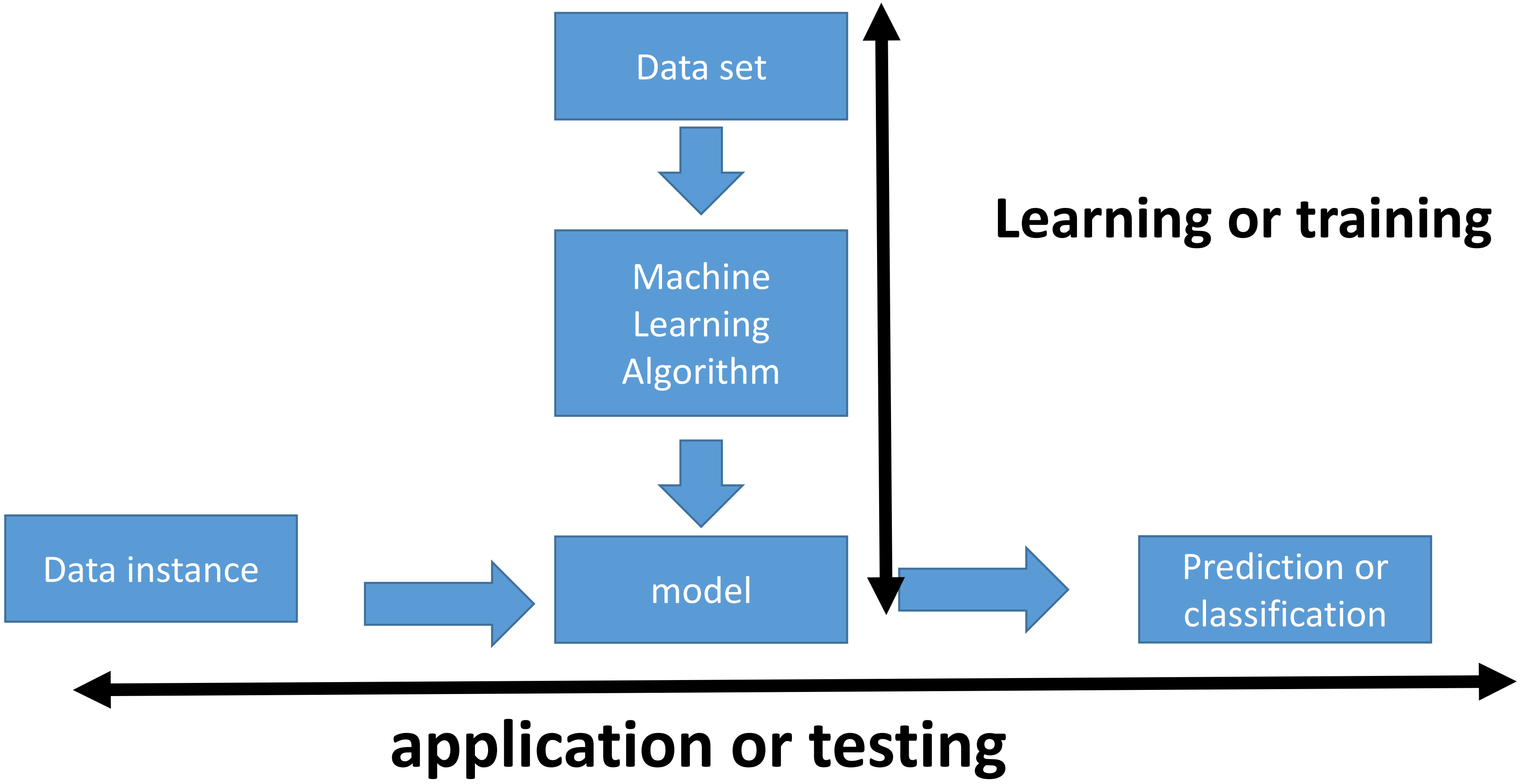
Aim:Take existing code and improve it automatically

- Applications
  - *Airport ground movements.*
  - *Software engineering*
  - *Medicine – heart disease indicator*

I currently teach on programme at BUPT – 3 years
Previously at University Nottingham Ningbo China – 4 years

http://gpbib.cs.ucl.ac.uk/gp-html/index.html (40th / 10,000. 2nd largest AI BIB)
https://scholar.google.co.uk/citations?user=iZIjJ80AAAAJ&hl=en

Supervised Machine Learning



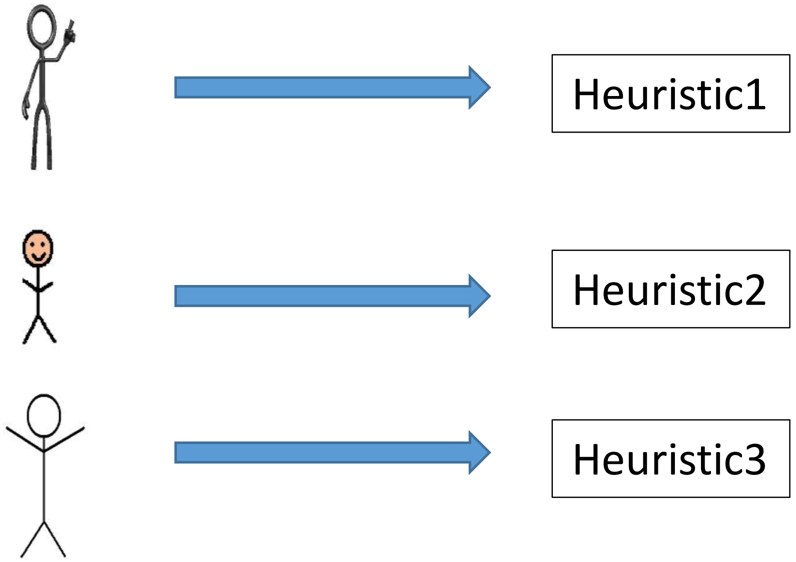
One Man – One/Many Algorithm

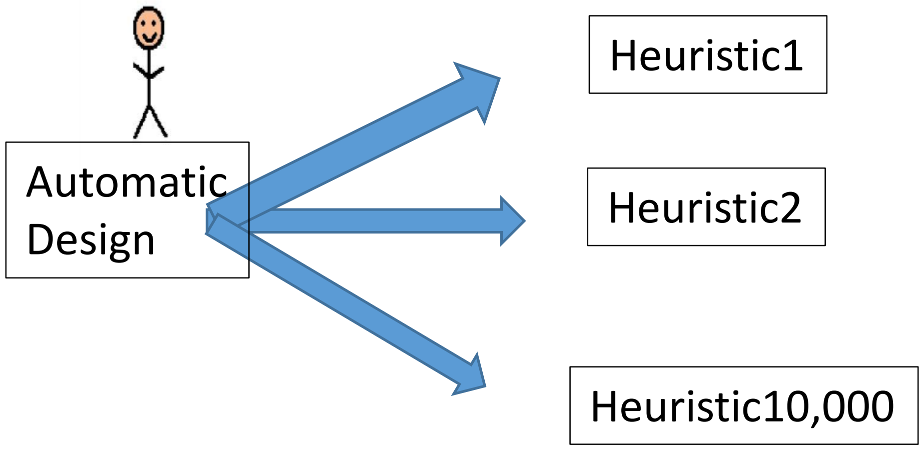
1. Researchers design heuristics by hand and test them on problem instances or arbitrary benchmarks off internet.
2. Presenting results at conferences and publishing in journals. In this talk/paper we propose a new algorithm…

1. Challenge is defining an algorithmic framework (**set**) that **includes** useful algorithms. **Black art**

2. Let Genetic Programming **select the best algorithm for the problem class at hand.** **Context!!!** Let the data speak for itself without imposing our assumptions.

In this talk/paper we propose a 10,000 algorithms…

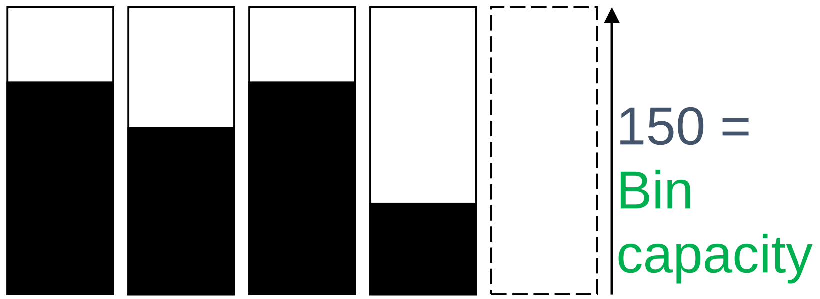




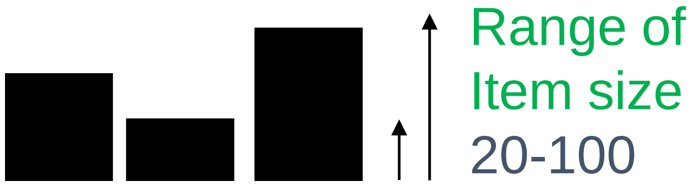
On-line Bin Packing Problem [9,11]

1. A *sequence* of items packed into as few a bins as possible.
2. Bin size is 150 units, items uniformly distributed between 20-100.
3. Different to the off-line bin packing problem where the *set* of items.
4. The “best fit” heuristic, places the current item in the space it fits best (leaving least slack).
5. It has the property that this heuristic does not open a new bin unless it is forced to.

Array of bins

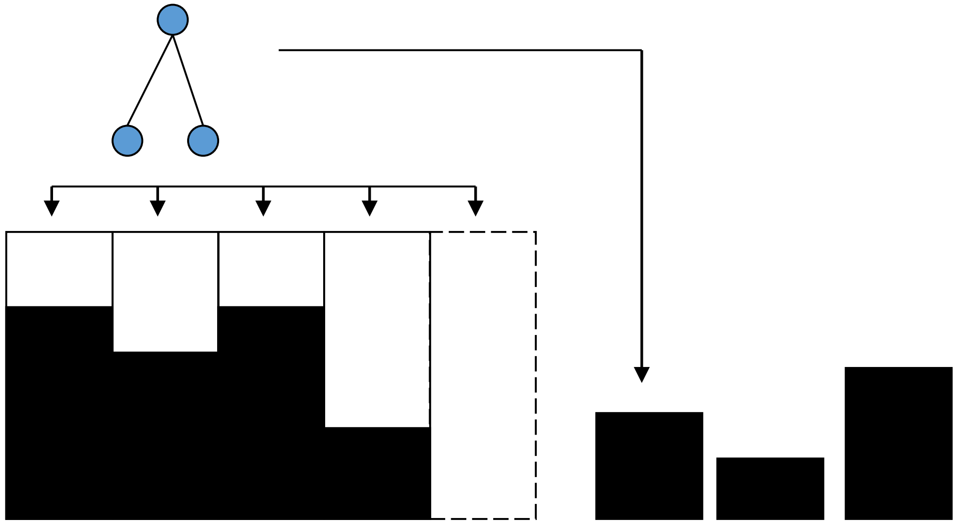


Items packed so far

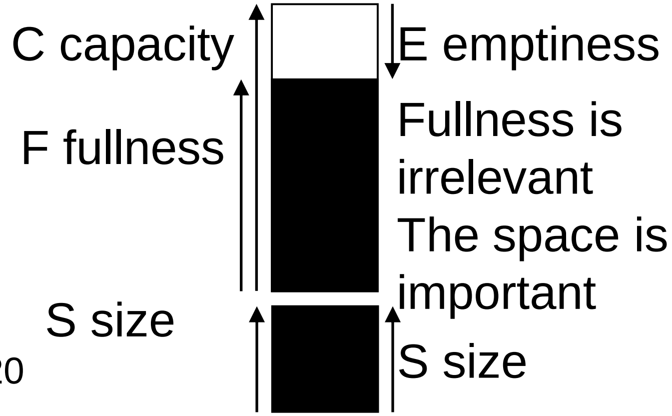


Sequence of pieces to be packed

Genetic Programming applied to on-line bin packing

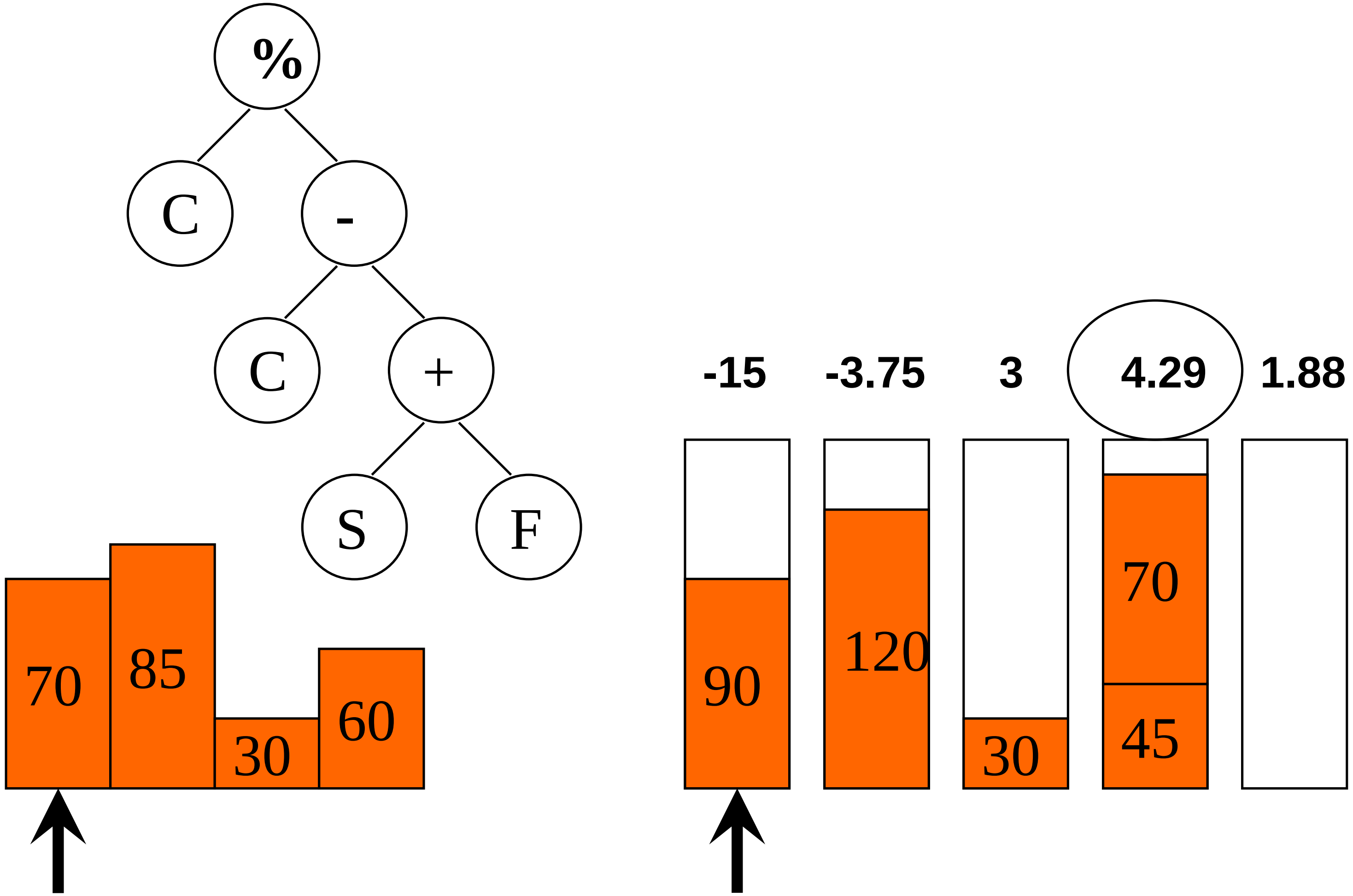


Not obvious how to link Genetic Programming to combinatorial problems.
The GP tree is applied to each bin with the current item and placed in the bin with
The maximum score



Terminals supplied to Genetic Programming
Initial representation {C, F, S}
Replaced with {E, S}, E=C-F

How the heuristics are applied (skip)



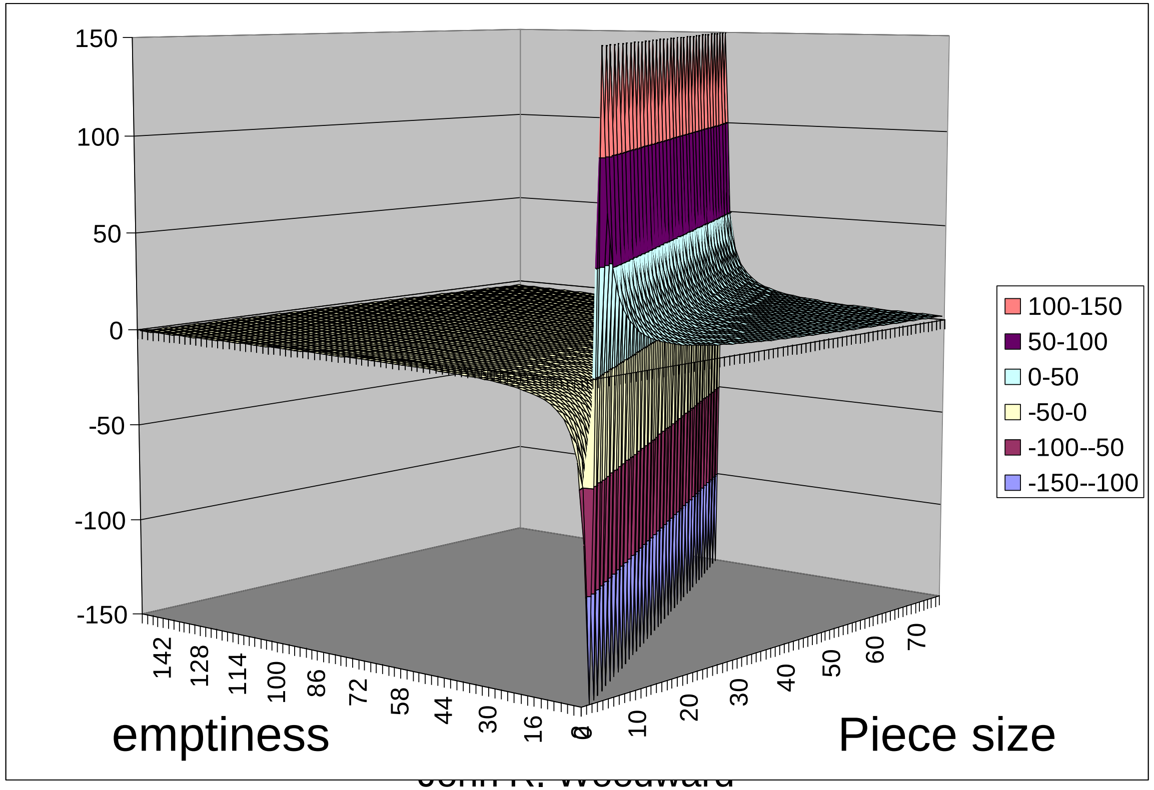
The Best Fit Heuristic

Best fit = 1/(E-S). Point out features.

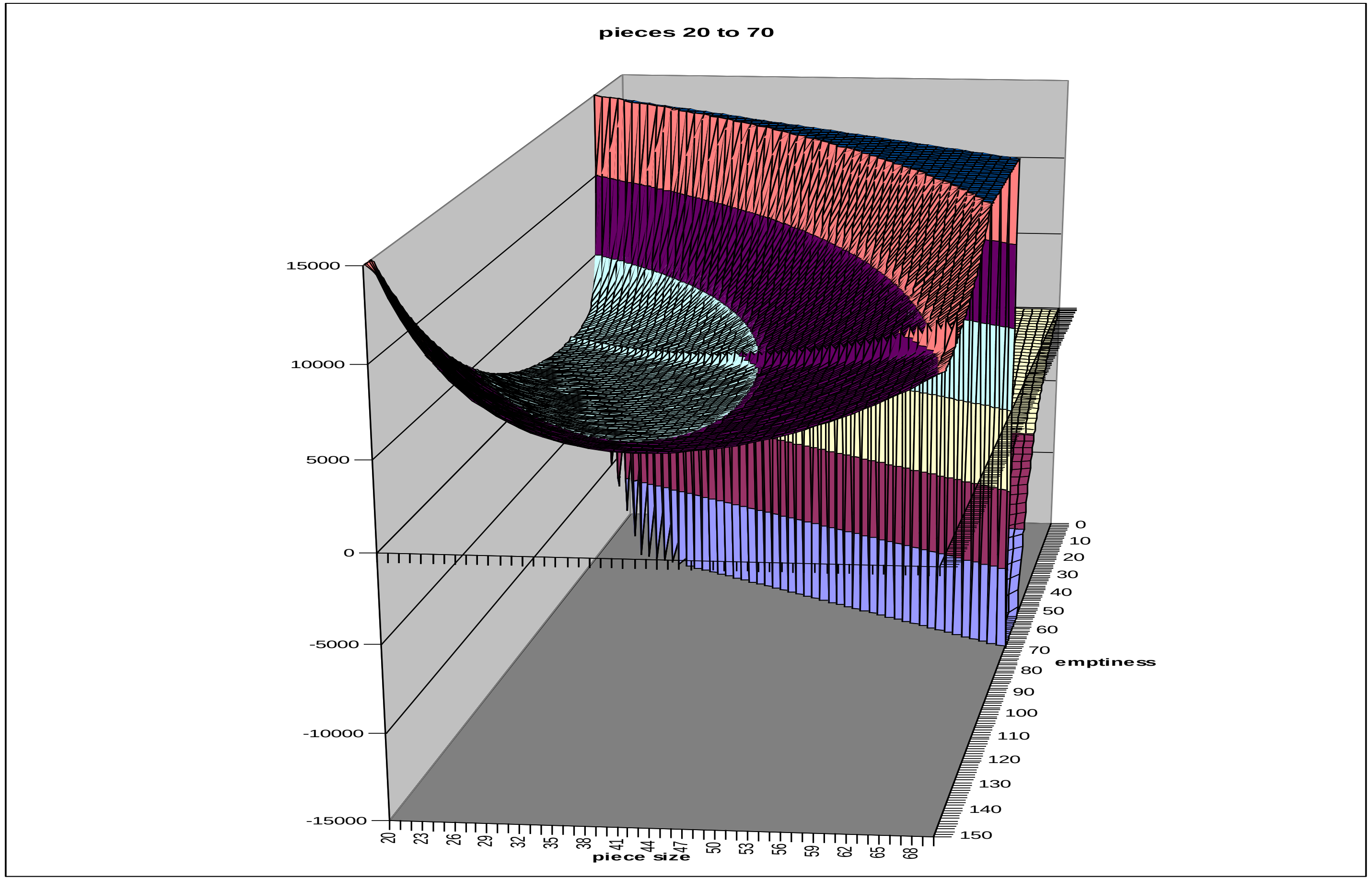
Pieces of size S, which fit well into the space remaining E, score well.

Best fit applied produces a set of points on the surface,

The bin corresponding to the maximum score is picked.

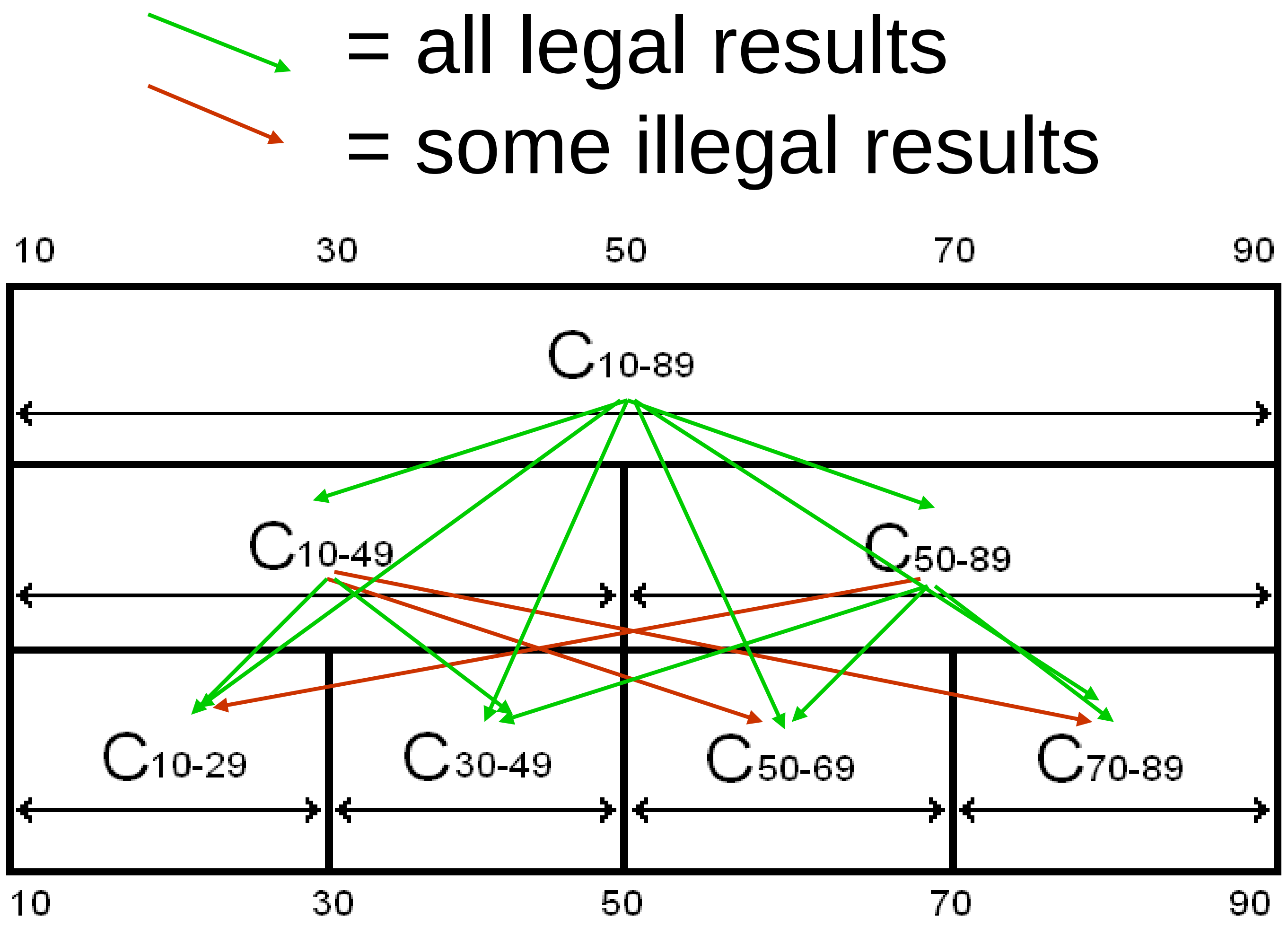


Our best heuristic.



Similar shape to best fit – but curls up in one corner.
Note that this is rotated, relative to previous slide.

Robustness of Heuristics



Testing Heuristics on problems of much larger size than in training

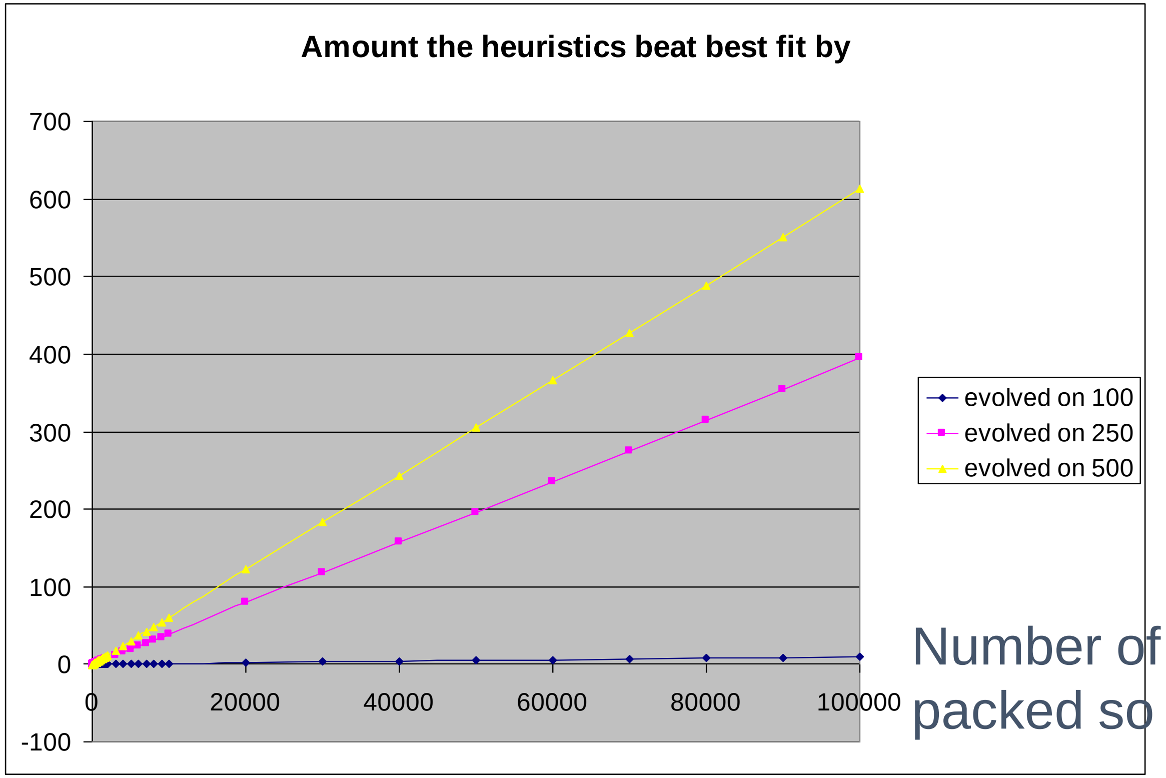
Table I	*H trained100*	*H trained 250*	*H trained 500*
100	0.427768358	0.298749035	0.140986023
1000	0.406790534	0.010006408	0.000350265
10000	0.454063071	2.58E-07	9.65E-12
100000	0.271828318	1.38E-25	2.78E-32

Table shows p-values using the best fit heuristic, for heuristics trained on different size problems, when applied to different sized problems

1. As number of items trained on increases, the probability decreases (see next slide).
2. As the number of items packed increases, the probability decreases (see next slide).

Compared with Best Fit

Amount evolved heuristics beat best fit by.

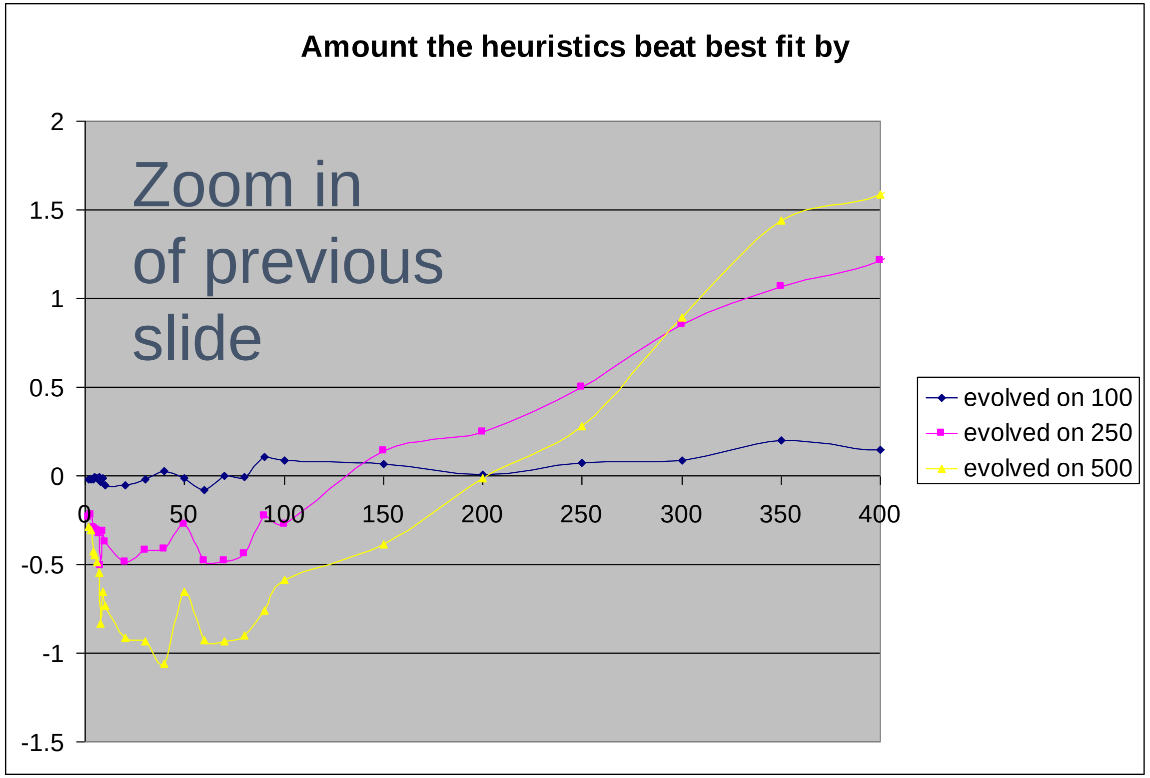


Number of pieces packed so far.

- Averaged over 30 heuristics over 20 problem instances
- Performance does not deteriorate
- The larger the training problem size, the better the bins are packed.

Compared with Best Fit

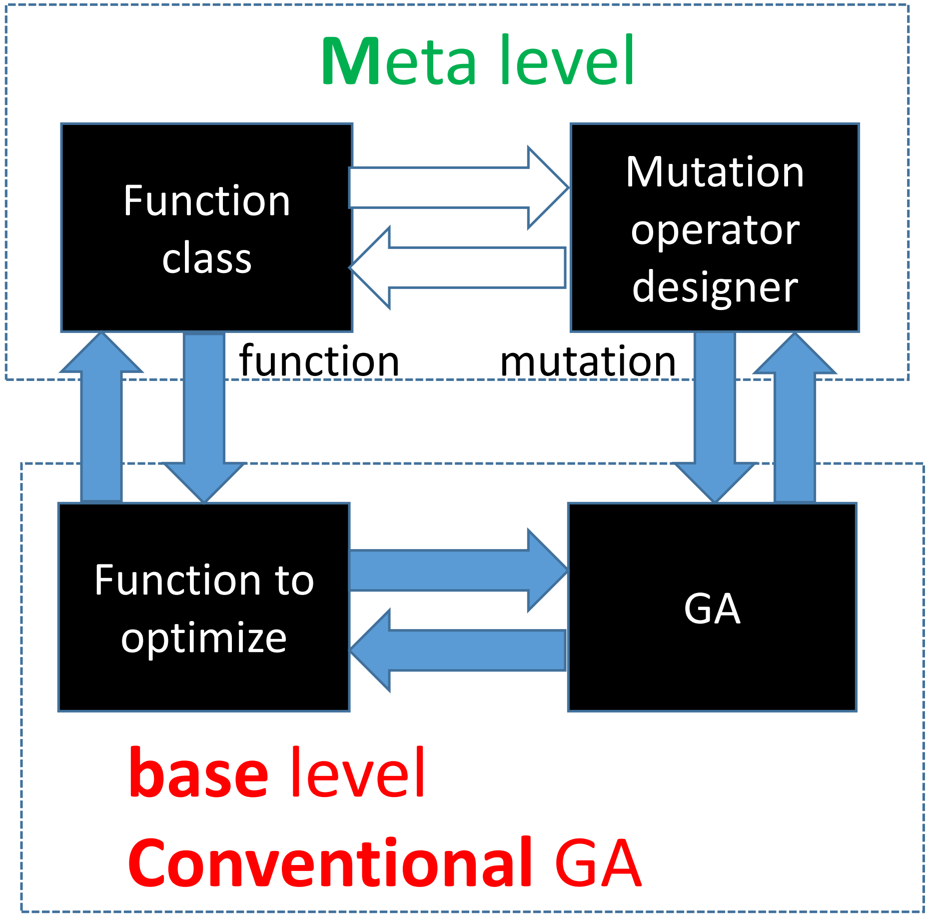
Amount evolved heuristics beat best fit by.



- The heuristic seems to learn the number of pieces in the problem
- Analogy with sprinters running a race – accelerate towards end of race.
- The “break even point” is approximately half of the size of the training problem size
- If there is a gap of size 30 and a piece of size 20, it would be better to wait for a better piece to come along later – about 10 items (similar effect at upper bound?).

Meta and Base Learning [15]

1. At the **base** level we are learning about a **specific** function.
2. At the **meta** level we are learning about the probability distribution.
3. We are just doing **"generate and test"** on **"generate and test"**
4. What is being passed with each **blue arrow**?
5. Training/Testing and Validation



Compare Signatures (Input-Output)

<u>Genetic Algorithm</u>

- $(B^n \rightarrow R) \rightarrow B^n$

Input is an objective function mapping bit-strings of length n to a real-value.

Output is a (near optimal) bit-string

i.e. the <u>solution</u> to the problem <u>instance</u>

<u>Genetic Algorithm FACTORY</u>

- $[(B^n \rightarrow R)] \rightarrow ((B^n \rightarrow R) \rightarrow B^n)$

Input is a *list of* functions mapping bit-strings of length n to a real-value (i.e. sample problem instances from the problem class).

Output is a (near optimal) mutation operator for a GA

i.e. the <u>solution</u> <u>method</u> (algorithm) to the <u>problem</u> <u>class</u>

We are raising the level of generality at which we operate.

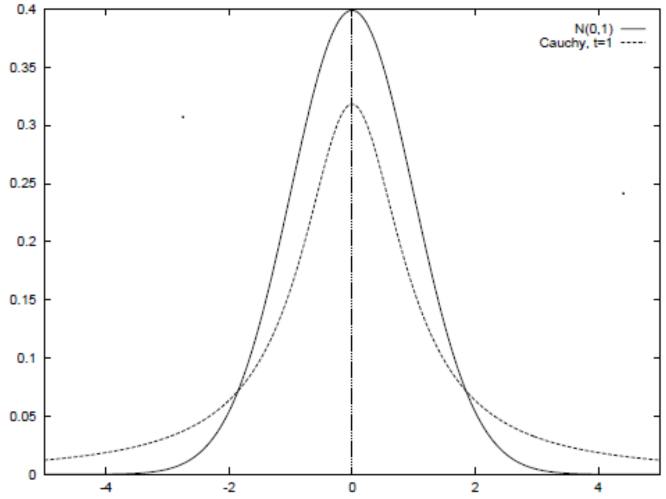
Designing Mutation Operators for Evolutionary Programming [18]

1. **Evolutionary programing** optimizes functions by evolving a population of real-valued vectors (genotype).
2. **Variation** has been provided (manually) by **probability distributions** (**Gaussian, Cauchy, Levy**).
3. We are **automatically generating** probability distributions (using genetic programming).
4. **Not from scratch**, but from already well known distributions (**Gaussian, Cauchy, Levy**). We are "**genetically improving probability distributions**".
5. We are evolving mutation operators **for a problem class** (a probability distributions over functions).
6. NO CROSSOVER

Genotype is

(1.3,...,4.5,...,8.7)

Before mutation



Genotype is

(1.2,...,4.4,...,8.6)

After mutation

(Fast) Evolutionary Programming

Heart of algorithm is mutation
SO LETS AUTOMATICALLY DESIGN

$$x_i'(j) = x_i(j) + \eta_i(j)D_j$$

1. **EP** mutates with a **Gaussian**
2. **FEP** mutates with a **Cauchy**
3. A **generalization** is mutate with a **distribution D** (generated with genetic programming)

1. Generate the initial population of μ individuals, and set $k = 1$. Each individual is taken as a pair of real-valued vectors, (x_i, η_i), $\forall i \in \{1, \cdots, \mu\}$.
2. Evaluate the fitness score for each individual (x_i, η_i), $\forall i \in \{1, \cdots, \mu\}$, of the population based on the objective function, $f(x_i)$.
3. Each parent (x_i, η_i), $i = 1, \cdots, \mu$, creates a single offspring (x_i', η_i') by: for $j = 1, \cdots, n$,

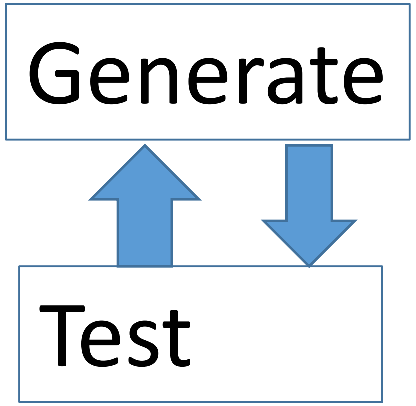
$$x_i'(j) = x_i(j) + \eta_i(j)N(0,1), \quad (1)$$
$$\eta_i'(j) = \eta_i(j)\exp(\tau' N(0,1) + \tau N_j(0,1)) \quad (2)$$

   where $x_i(j)$, $x_i'(j)$, $\eta_i(j)$ and $\eta_i'(j)$ denote the j-th component of the vectors x_i, x_i', η_i and η_i', respectively. $N(0,1)$ denotes a normally distributed one-dimensional random number with mean zero and standard deviation one. $N_j(0,1)$ indicates that the random number is generated anew for each value of j. The factors τ and τ' have commonly set to $\left(\sqrt{2\sqrt{n}}\right)^{-1}$ and $\left(\sqrt{2n}\right)^{-1}$ [9, 8].
4. Calculate the fitness of each offspring (x_i', η_i'), $\forall i \in \{1, \cdots, \mu\}$.
5. Conduct pairwise comparison over the union of parents (x_i, η_i) and offspring (x_i', η_i'), $\forall i \in \{1, \cdots, \mu\}$. For each individual, q opponents are chosen randomly from all the parents and offspring with an equal probability. For each comparison, if the individual's fitness is no greater than the opponent's, it receives a "win."
6. Select the μ individuals out of (x_i, η_i) and (x_i', η_i'), $\forall i \in \{1, \cdots, \mu\}$, that have the most wins to be parents of the next generation.
7. Stop if the stopping criterion is satisfied; otherwise, $k = k + 1$ and go to Step 3.

Evolution GA/GP

- Generate and test: cars, code, models, proofs, medicine, hypothesis.
- Evolution (select, vary, inherit).
- Fit for purpose

Feedback loop
Humans
Computers



Inheritance
Off-spring have similar Genotype (phenotype)
PERFECT CODE [3]

Optimization & Benchmark Functions

A set of 23 benchmark functions is typically used in the literature. **Minimization** $\forall x \in S : f(x_{min}) \leq f(x)$

We use them as **problem classes**.

Table 1: The 23 test functions used in our experimental studies, where n is the dimension of the function, f_{min} the minimum value of the function, and $S \subseteq R^n$.

Test function	n	S	f_{min}
$f_1(x) = \sum_{i=1}^{n} x_i^2$	30	$[-100, 100]^n$	0
$f_2(x) = \sum_{i=1}^{n} \lvert x_i \rvert + \prod_{i=1}^{n} \lvert x_i \rvert$	30	$[-10, 10]^n$	0
$f_3(x) = \sum_{i=1}^{n} (\sum_{j=1}^{i} x_j)^2$	30	$[-100, 100]^n$	0
$f_4(x) = \max_i\{\lvert x_i \rvert, 1 \leq i \leq n\}$	30	$[-100, 100]^n$	0
$f_5(x) = \sum_{i=1}^{n-1} [100(x_{i+1} - x_i^2)^2 + (x_i - 1)^2]$	30	$[-30, 30]^n$	0
$f_6(x) = \sum_{i=1}^{n} \lfloor x_i + 0.5 \rfloor$	30	$[-100, 100]^n$	0
$f_7(x) = \sum_{i=1}^{n} i x_i^4 + random[0, 1)$	30	$[-1.28, 1.28]^n$	0
$f_8(x) = \sum_{i=1}^{n} -x_i \sin(\sqrt{\lvert x_i \rvert})$	30	$[-500, 500]^n$	-12569.5
$f_9(x) = \sum_{i=1}^{n} [x_i^2 - 10\cos(2\pi x_i) + 10)]$	30	$[-5.12, 5.12]^n$	0
$f_{10}(x) = -20\exp\left(-0.2\sqrt{\frac{1}{n}\sum_{i=1}^{n} x_i^2}\right) - \exp\left(\frac{1}{n}\sum_{i=1}^{n} \cos 2\pi x_i\right) + 20 + e$	30	$[-32, 32]^n$	0

Function Class 1

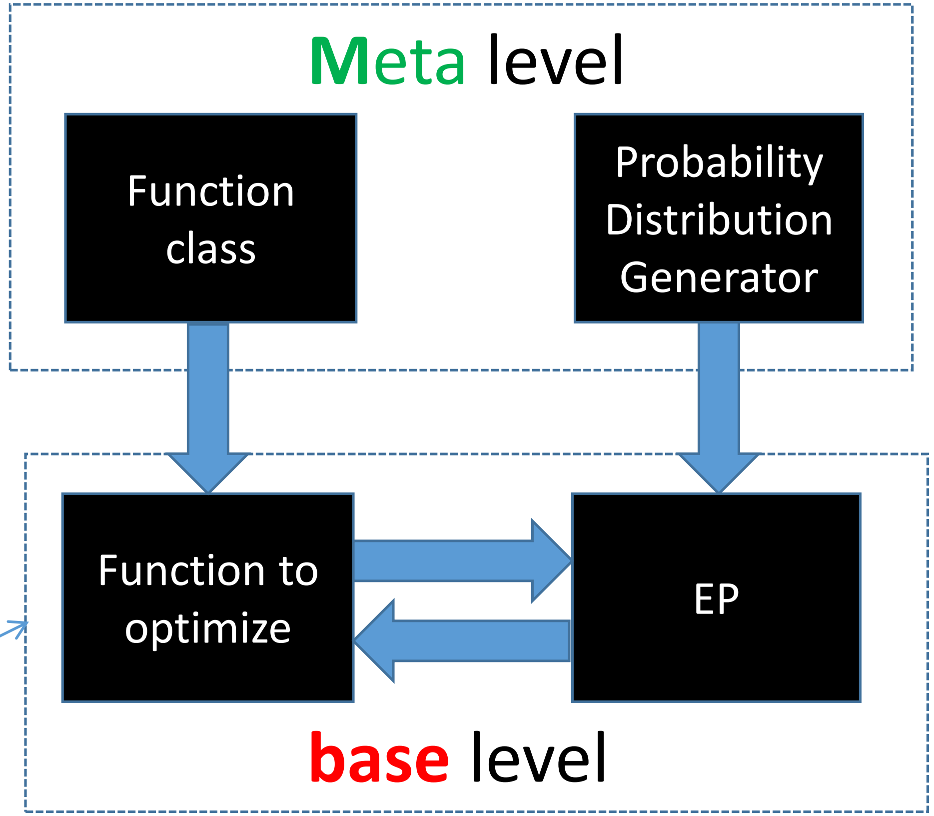
1. Machine learning needs to generalize.
2. We generalize to function classes.
3. y = x^2 (**a function**)
4. y = ax^2(parameterised function)
5. y = ax^2, a ~[1,2] (**function class**)
6. We do this for all benchmark functions.
7. **The mutation operators is evolved to fit the probability distribution of functions**.

Function Classes 2

$Function\ Classes$	S	b	f_{min}
$f_1(x) = a\sum_{i=1}^{n} x_i^2$	$[-100,100]^n$	N/A	0
$f_2(x) = a\sum_{i=1}^{n} \mid x_i \mid + b\prod_{i=1}^{n} \mid x_i \mid$	$[-10,10]^n$	$b \in [0, 10^{-5}]$	0
$f_3(x) = \sum_{i=1}^{n}(a\sum_{j=1}^{i} x_j)^2$	$[-100,100]^n$	N/A	0
$f_4(x) = \max_i\{a \mid x_i \mid, 1 \leq i \leq n\}$	$[-100,100]^n$	N/A	0
$f_5(x) = \sum_{i=1}^{n}[a(x_{i+1} - x_i^2)^2 + (x_i - 1)^2]$	$[-30,30]^n$	N/A	0
$f_6(x) = \sum_{i=1}^{n}(\lfloor ax_i + 0.5 \rfloor)^2$	$[-100,100]^n$	N/A	0
$f_7(x) = a\sum_{i=1}^{n} ix_i^4 + random[0,1)$	$[-1.28,1.28]^n$	N/A	0
$f_8(x) = \sum_{i=1}^{n} -(x_i \sin(\sqrt{\mid x_i \mid}) + a)$	$[-500,500]^n$	N/A	[-12629.5, -12599.5]
$f_9(x) = \sum_{i=1}^{n}[ax_i^2 + b(1 - cos(2\pi x_i))]$	$[-5.12,5.12]^n$	$b \in [5,\ 10]$	0
$f_{10}(x) = -a\exp(-0.2\sqrt{\frac{1}{n}\sum_{i=1}^{n} x_i^2}) - \exp(\frac{1}{n}\sum_{i=1}^{n} \cos 2\pi x_i) + a + e$	$[-32,32]^n$	N/A	0

Meta and Base Learning

- At the **base** level we are learning about a **specific** function.
- At the **meta** level we are learning about the problem **class**.
- We are just doing **"generate and test"** at a higher level
- What is being passed with each **blue arrow**?
- **Conventional** EP



Compare Signatures (Input-Output)

<u>Evolutionary Programming</u>

$(R^n \rightarrow R) \rightarrow R^n$

Input is a function mapping real-valued vectors of length n to a real-value.

Output is a (near optimal) real-valued vector

(i.e. the <u>solution</u> to the problem <u>instance</u>)

<u>Evolutionary Programming Designer</u>

$[(R^n \rightarrow R)] \rightarrow ((R^n \rightarrow R) \rightarrow R^n)$

Input is a *list of* functions mapping real-valued vectors of length n to a real-value (i.e. sample problem instances from the problem class).

Output is a (near optimal) (mutation operator for) Evolutionary Programming

(i.e. the <u>solution</u> <u>method</u> to the problem <u>class</u>)

We are **raising the level of generality** at which we operate.

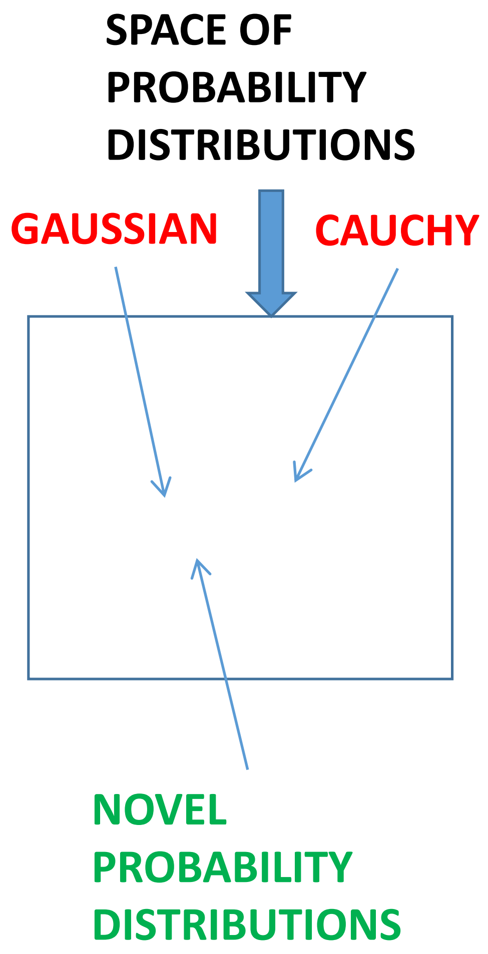
Genetic Programming to Generate Probability Distributions

1. GP **Function Set** {+, -, *, %}
2. GP **Terminal Set** {N(0, random)}

N(0,1) is a normal distribution.

For example a Cauchy distribution is generated by N(0,1)%N(0,1).

Hence **the search space of probability distributions** contains the two existing probability distributions used in EP but also **novel probability distributions**.



Means and Standard Deviations

These results are good for two reasons.

1. starting with a manually designed distributions (Gaussian).

2. evolving distributions **for each function class**.

Function Class	FEP Mean Best	FEP Std Dev	CEP Mean Best	CEP Std Dev	GP-distribution Mean Best	GP-distribution Std Dev
f_1	1.24×10^{-3}	2.69×10^{-4}	1.45×10^{-4}	9.95×10^{-5}	$\mathbf{6.37\times10^{-5}}$	5.56×10^{-5}
f_2	1.53×10^{-1}	2.72×10^{-2}	4.30×10^{-2}	9.08×10^{-3}	$\mathbf{8.14\times10^{-4}}$	8.50×10^{-4}
f_3	2.74×10^{-2}	2.43×10^{-2}	5.15×10^{-2}	9.52×10^{-2}	$\mathbf{6.14\times10^{-3}}$	8.78×10^{-3}
f_4	1.79	1.84	1.75×10	6.10	$\mathbf{2.16\times10^{-1}}$	6.54×10^{-1}
f_5	2.52×10^{-3}	4.96×10^{-4}	2.66×10^{-4}	4.65×10^{-5}	$\mathbf{8.39\times10^{-7}}$	1.43×10^{-7}
f_6	3.86×10^{-2}	3.12×10^{-2}	4.40×10	1.42×10^{2}	$\mathbf{9.20\times10^{-3}}$	1.34×10^{-2}
f_7	6.49×10^{-2}	1.04×10^{-2}	6.64×10^{-2}	1.21×10^{-2}	$\mathbf{5.25\times10^{-2}}$	8.46×10^{-3}
f_8	-11342.0	3.26×10^{2}	-7894.6	6.14×10^{2}	**-12611.6**	2.30×10
f_9	6.24×10^{-2}	1.30×10^{-2}	1.09×10^{2}	3.58×10	$\mathbf{1.74\times10^{-3}}$	4.25×10^{-4}
f_{10}	1.67	4.26×10^{-1}	1.45	2.77×10^{-1}	**1.38**	2.45×10^{-1}

T-tests

Table 5 2-tailed t-tests comparing EP with GP-distributions, FEP and CEP on f_1-f_{10}.

Function Class	Number of Generations	GP-distribution vs FEP *t-test*	GP-distribution vs CEP *t-test*
f_1	1500	2.78×10^{-47}	4.07×10^{-2}
f_2	2000	5.53×10^{-62}	1.59×10^{-54}
f_3	5000	8.03×10^{-8}	1.14×10^{-3}
f_4	5000	1.28×10^{-7}	3.73×10^{-36}
f_5	20000	2.80×10^{-58}	9.29×10^{-63}
f_6	1500	1.85×10^{-8}	3.11×10^{-2}
f_7	3000	3.27×10^{-9}	2.00×10^{-9}
f_8	9000	7.99×10^{-48}	5.82×10^{-75}
f_9	5000	6.37×10^{-55}	6.54×10^{-39}
f_{10}	1500	9.23×10^{-5}	1.93×10^{-1}

Performance on Other Problem Classes

Table 8: This table compares the fitness values (averaged over 20 runs) of each of the 23 ADRs on each of the 23 function classes. Stardard deviations are in parentheses.

	ADR1	ADR2	ADR3	ADR4	ADR5	ADR6	ADR7	ADR8	ADR9	ADR10	ADR11	ADR12	ADR13	ADR14	ADR15	ADR16	ADR17	ADR18	ADR19	ADR20	ADR21	ADR22	ADR23
f_1																							
f_2																							
f_3																							
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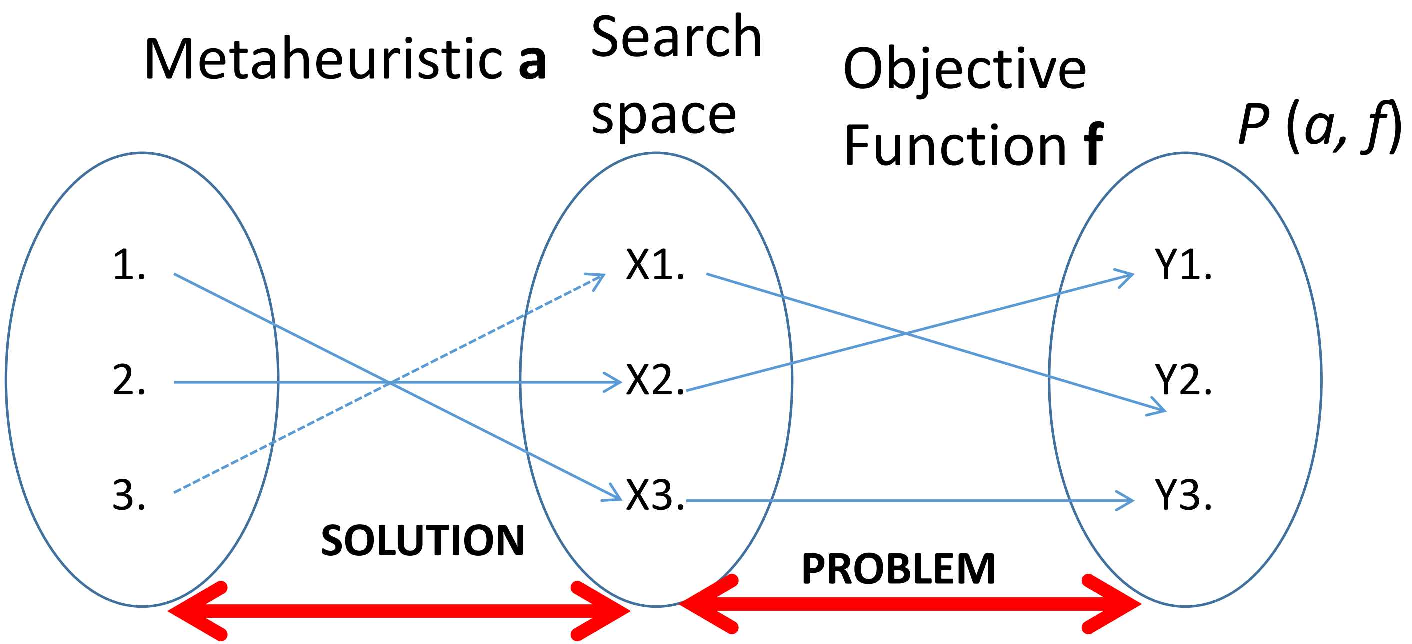
Performance on Other Problem Classes

Table 8: This table compares the fitness values (averaged over 20 runs) of each of the 23 ADRs on each of the 23 function classes. Stardard deviations are in parentheses.

	ADR1	ADR2	ADR3	ADR4	ADR5	ADR6	ADR7	ADR8	ADR9	ADR10	ADR11	ADR12
f_1	**3.796042378**	3.796795988	3.796172495	3.800097053	3.798215201	3.798426888	3.79682023	2531.207552	3.796057571	**3.795990923**	3.845998009	3.796186664
	(15.53395359)	(15.53405343)	(15.53391881)	(15.53360785)	(15.53375201)	(15.53375598)	(15.53387687)	(11290.25839)	(15.53388934)	(15.53393945)	(15.52984118)	(15.533912)
f_2	0.03272533	**0.017265507**	0.06411854	0.243765938	0.227029932	0.242415867	0.141152255	10.89831484	**0.012640918**	**0.015574102**	0.878284278	0.048279715
	(0.006220059)	(0.00316433)	(0.012214985)	(0.052833925)	(0.041526229)	(0.045421712)	(0.027254796)	(29.61351836)	(0.002366678)	(0.003232023)	(0.224644483)	(0.009029109)
f_3	0.059115953	0.542598852	**0.018626247**	0.103132914	**0.013948627**	**0.014236266**	**0.005922877**	33.77718802	2.665366951	2.039338209	0.937932626	0.28659926
	(0.127613356)	(0.907149879)	(0.038845313)	(0.052212998)	(0.006159356)	(0.005987797)	(0.003058048)	(25.60578768)	(3.027285258)	(2.680462701)	(0.471271529)	(0.420720664)
f_4	17.25966091	19.41813374	16.15346434	**0.292628803**	9.648461377	14.04875432	16.79279446	28.13359508	10.99661065	14.19501939	**0.162317701**	2.630226876
	(4.966599608)	(7.253429181)	(4.887330993)	(0.502787856)	(5.399453121)	(7.007590006)	(7.260923198)	(43.09163947)	(5.899934832)	(6.765307096)	(0.125141891)	(2.042641555)
f_5	-12.59873403	-11.39294662	**-13.38616242**	-12.17766112	**-12.98617719**	**-13.26385156**	**-13.4343626**	4522608.395	-12.80651566	-12.38416526	-10.21081863	-12.61795619
	(19.08466205)	(16.77875418)	(19.29489759)	(20.17833659)	(18.94371583)	(19.30900217)	(19.83357655)	(4559249.522)	(19.38895811)	(17.82800791)	(18.25643711)	(18.513)
f_6	422.0335	1143.0035	12.165	0.135	0.058	**0.0535**	0.11	9.298	0.107	**0.015**	0.3255	**0.039**
	(1649.46925)	(2074.280164)	(38.34787739)	(0.264724286)	(0.221943568)	(0.222858818)	(0.304613647)	(6.862988915)	(0.25034923)	(0.022826577)	(0.157228463)	(0.061034934)
f_7	0.065542671	0.078308367	0.056194615	0.08411716	**0.047030838**	0.04882212	**0.048746647**	2.807243879	0.063532032	0.058626761	0.194904423	0.061920648
	(0.010787905)	(0.024299068)	(0.011619042)	(0.019571033)	(0.00605618)	(0.00653602)	(0.0077357)	(1.21388786)	(0.012804765)	(0.009280782)	(0.041805987)	(0.010715646)
f_8	-7476.802093	-7873.512071	-8371.08948	-11531.07729	-8638.189529	-8567.792279	-8461.852799	**-12471.54499**	-10836.75693	-10864.24065	-11921.10551	-11261.25295
	(648.3654776)	(503.1138893)	(779.6316972)	(307.0584479)	(533.7011029)	(489.8047705)	(682.1430256)	(158.4399895)	(498.9742452)	(528.4915592)	(319.6841114)	(332.9818897)
f_9	-6.364340621	-6.613834318	-7.274192496	-8.841127079	-6.525185945	-6.624821944	-6.117765239	-7.490475004	**-8.854034752**	-8.854016567	-8.724342579	-8.853598499
	(13.55211805)	(14.20272875)	(13.10504988)	(11.27787199)	(13.91130284)	(14.11686839)	(14.36718746)	(11.30142186)	(11.28014913)	(11.28014763)	(11.2529754)	(11.28005901)
f_{10}	-26.54875358	-27.62462039	-27.87344964	-28.52972749	-28.43776411	-27.47491154	-28.12468594	-7.253656564	**-28.58283002**	**-28.58195607**	-23.84876736	-28.57454432
	(8.642935672)	(7.694296506)	(7.11115815)	(6.280710263)	(6.447779575)	(8.117435238)	(6.629475271)	(14.70586537)	(6.292607345)	(6.292379922)	(13.2608428)	(6.290797959)
f_{11}	-0.651086092	-0.430374859	-0.699398783	-0.732192039	-0.696833115	-0.722930189	-0.681421101	-0.715900311	-0.640236618	-0.674184211	**-0.738435445**	-0.717184562
	(0.553088246)	(0.589562784)	(0.53391987)	(0.551795141)	(0.565782865)	(0.543780683)	(0.546113958)	(0.545831558)	(0.558785685)	(0.546230838)	(0.548435173)	(0.564381489)
f_{12}	2.375745683	1.519931686	0.81756969	0.000049864	1.138872739	1.081912616	2.613131902	295302505.2	0.086323699	0.048961716	0.000761494	**0.000002742**
	(2.850960856)	(2.154711219)	(1.297295505)	(1.32181E-05)	(1.204159184)	(2.165644097)	(3.63485628)	(562235782.9)	(0.29953258)	(0.176719921)	(0.000235795)	(1.09208E-06)
f_{13}	7.528102918	11.54428374	5.380220018	0.000668631	3.178046205	5.343480627	10.81477842	451382341.6	0.003800314	0.001868601	0.010121716	**4.50175E-05**
	(11.50730323)	(12.5903624)	(10.27590186)	(0.000138084)	(5.035811692)	(5.584041018)	(19.74462848)	(611339756.7)	(0.007130406)	(0.004563481)	(0.002309003)	(2.01955E-05)
f_{14}	1.734470721	1.772821551	1.230172967	1.072006527	1.693127216	1.806118977	1.165167477	**0.754781615**	1.292947098	1.048015299	**0.780822999**	**0.833866789**
	(1.392570373)	(1.516523557)	(0.555507449)	(0.655936932)	(1.661752265)	(1.345479048)	(0.596149785)	(0.17253071)	(1.07692978)	(0.572821944)	(0.174823651)	(0.309901545)
f_{15}	**0.000643324**	0.001546119	0.000784565	0.000765787	0.002250898	0.001921141	0.001197892	0.001639261	**0.000535974**	**0.000417704**	0.001101053	**0.000580249**
	(0.000543038)	(0.004444802)	(0.00086432)	(0.000855092)	(0.0058185)	(0.003532418)	(0.001824797)	(0.001952972)	(0.000372644)	(0.000285176)	(0.001179448)	(0.000400097)
f_{16}	-1.92447213	-1.924473263	-1.924470875	-1.924468207	-1.924439114	-1.924429001	-1.924455664	-1.924280681	-1.924473516	-1.924473529	-1.924454275	-1.924473348
	(0.579460448)	(0.579460606)	(0.579460481)	(0.579458449)	(0.579459593)	(0.579458044)	(0.579458793)	(0.57965821)	(0.579460635)	(0.579460628)	(0.579457791)	(0.579460625)
f_{17}	3.786952682	3.786952414	3.786953089	3.786953821	3.786960153	3.786965855	3.786956682	3.787007025	**3.786952351**	**3.786952347**	3.78696211	3.786952403
	(2.374931079)	(2.374931155)	(2.374931035)	(2.374930466)	(2.374929682)	(2.374928026)	(2.374930239)	(2.374940369)	(2.374931165)	(2.374931168)	(2.374931608)	(2.374931147)
f_{18}	4.575031533	**4.574971819**	4.575098811	4.575282035	4.5763558	4.577539016	4.575858015	4.588715133	**4.574958404**	**4.574958205**	4.576371722	**4.574967813**
	(1.117668123)	(1.117658005)	(1.117699009)	(1.117651066)	(1.117796611)	(1.118208418)	(1.117843077)	(1.112380559)	(1.117656673)	(1.117656611)	(1.118018727)	(1.11765998)
f_{19}	-3.552067105	-3.552080968	-3.552050408	-3.551996944	-3.551673503	-3.538670434	-3.551882996	-3.55009538	-3.552083616	-3.552083593	-3.551749707	-3.55208093
	(1.903081577)	(1.903089133)	(1.903075519)	(1.903055044)	(1.902886076)	(1.915377418)	(1.903020205)	(1.901950912)	(1.903090389)	(1.903090534)	(1.902932325)	(1.903089578)

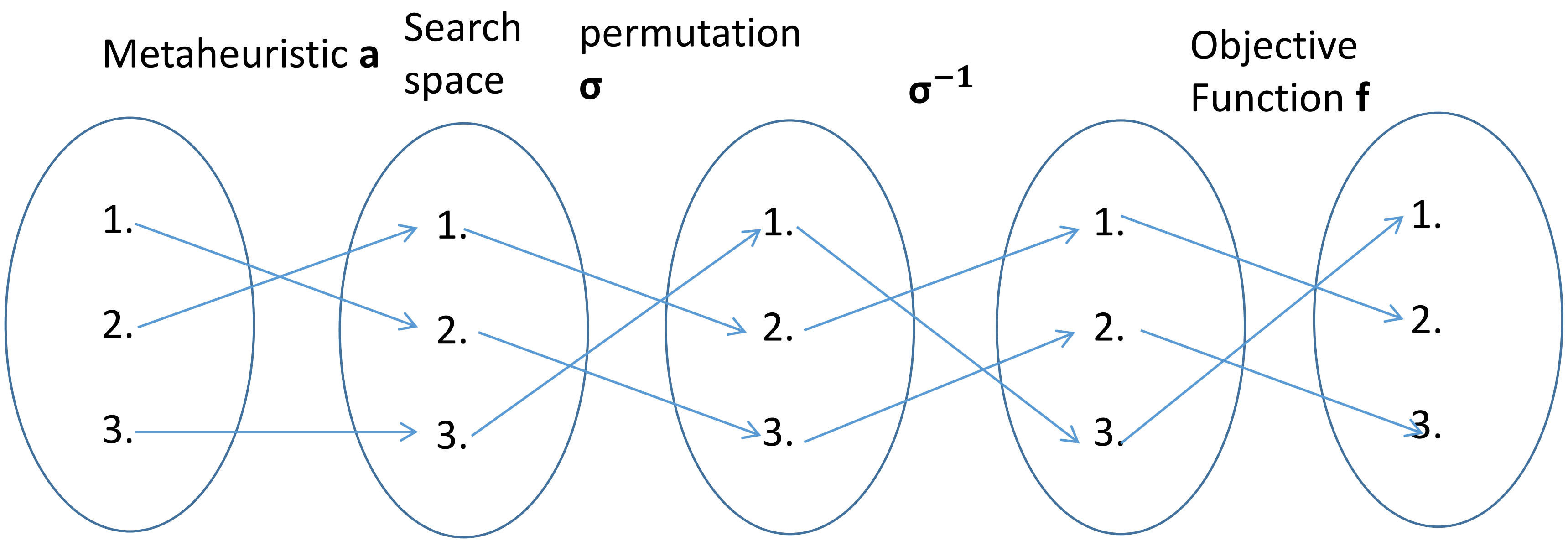
	ADR13	ADR14	ADR15	ADR16	ADR17	ADR18	ADR19	ADR20	ADR21	ADR22	ADR23
f_1	3.796277155	3.796649368	3.79612197	1655.307735	16670.67898	3.802819106	4.260445107	3.796097541	44.18750067	3.796066481	3.816827509
	(15.53392309)	(15.53392525)	(15.53399083)	(2395.677635)	(7873.980089)	(15.53117437)	(15.5103041)	(15.53398613)	(49.93392927)	(15.5339502)	(15.53473388)
f_2	0.068163286	0.10166228	0.033010725	5.652569538	62.08781058	0.02943591	**0.003081819**	**0.008143766**	0.269907184	0.033088529	0.039637186
	(0.013313298)	(0.021204407)	(0.006539326)	(6.56695685)	(15.77341379)	(0.005428489)	(0.000845787)	(0.001531136)	(0.148966146)	(0.006487538)	(0.038535293)
f_3	0.064134902	0.097019896	0.084879154	3433.708363	7087.024237	0.157436096	50.23471658	5.582568993	11.01739701	0.199225997	182.1465227
	(0.100019319)	(0.113519449)	(0.123340191)	(3872.306081)	(4298.4011)	(0.194641319)	(47.12256504)	(6.896844468)	(7.777401427)	(0.380877388)	(157.1263872)
f_4	4.399393447	1.330615099	21.57790808	68.40219553	74.66207074	40.88032128	37.07012251	16.70289914	61.04798152	15.80925637	8.929701211
	(2.852710021)	(1.700571261)	(7.946864347)	(7.994013124)	(7.257141818)	(9.606000218)	(14.00520825)	(5.748158881)	(8.858525787)	(4.977945429)	(5.603363277)
f_5	-12.07327907	-12.34060394	-11.37346075	768.5563656	189434.0968	-11.6837873	-10.20070261	-10.91325651	6.018008713	-12.79281207	-7.353638881
	(17.92596386)	(17.0817524)	(18.02353828)	(1491.144115)	(135909.7748)	(18.82095681)	(17.37621129)	(18.05348947)	(25.80280349)	(19.30159697)	(20.10064511)
f_6	**0.035**	**0.0315**	195.777	17987.881	37431.6995	424.4385	375.2155	**0.017**	4999.8915	185.43	83.996
	(0.066451407)	(0.024553915)	(490.1220371)	(13480.82092)	(19447.51813)	(752.3238003)	(888.0006481)	(0.025975697)	(5268.862537)	(446.2366362)	(123.8021751)
f_7	0.053685635	0.062253766	0.070244942	0.532824769	45.5222079	0.091505903	0.0869747	0.06586313	0.478628757	0.06288902	0.186500061
	(0.008060171)	(0.010746513)	(0.010506748)	(0.461115144)	(19.50162907)	(0.022831626)	(0.025469636)	(0.012332483)	(0.175096278)	(0.015746188)	(0.111182366)
f_8	-11017.45819	-11363.51973	-7712.376423	-9103.974424	-7289.483153	-7716.478481	-11715.11843	-10802.04284	-7922.383673	-8271.935456	-12347.43418
	(352.2752319)	(396.6847647)	(673.4167468)	(629.0672603)	(588.0128451)	(519.3255199)	(392.4030967)	(383.8291898)	(534.7702565)	(686.2946167)	(208.9940377)
f_9	-8.853174849	-8.852121206	-5.713104239	-8.427859381	11.80045883	-6.427611891	-8.754991403	**-8.85405211**	-5.865365514	-7.252578007	-8.832262084
	(11.27996138)	(11.2798762)	(15.24928375)	(11.22098317)	(19.66542025)	(13.47596148)	(11.37032461)	(11.28015264)	(13.78071024)	(13.42757703)	(11.25921778)
f_{10}	-28.57126049	-28.56345724	-27.30007959	-16.86690042	-4.344066607	-26.58932429	-28.48336407	**-28.58356645**	-23.42001988	-27.41309828	-28.54008083
	(6.290015625)	(6.288416006)	(7.779335696)	(9.531070088)	(2.227440058)	(8.952959967)	(6.385591493)	(6.292680027)	(9.536175616)	(7.761631077)	(6.286327759)
f_{11}	-0.735309936	-0.725127919	-0.619405568	72.67346703	245.0964286	-0.378051882	-0.315772447	-0.585609275	5.001410223	-0.595487053	-0.732208916
	(0.544806443)	(0.54116403)	(0.534518486)	(74.07522252)	(108.7039575)	(0.519124016)	(0.641591441)	(0.539005548)	(4.930986743)	(0.539010124)	(0.548139483)
f_{12}	0.023901023	0.000009886	1.502825411	34.73265169	15022643.14	2.147365455	0.200806538	0.121246677	18.16207189	1.133574956	42557.38789
	(0.059797341)	(3.87911E-06)	(1.325423866)	(35.78526991)	(24577491.03)	(1.758359464)	(0.423450793)	(0.321512405)	(10.05733127)	(1.058239746)	(190322.4214)
f_{13}	**6.34555E-05**	0.00012633	3.661952338	333.0155182	7358392.064	18.23218457	0.715447776	0.028620163	257.2859935	5.854254043	527.9210413
	(1.65034E-05)	(2.92279E-05)	(4.589088775)	(304.2916941)	(4794487.129)	(21.70208849)	(1.571904151)	(0.071766746)	(118.5776707)	(7.478665087)	(2359.940113)
f_{14}	1.242364004	**0.838551326**	1.463164304	2.630454628	0.99349036	1.348920833	1.250299666	1.651843912	1.535512059	1.045701707	0.871834862
	(1.224806407)	(0.344005191)	(0.857558242)	(3.711827934)	(0.547260035)	(0.870950738)	(1.992292006)	(1.303367715)	(1.417919709)	(0.683565682)	(0.346950096)
f_{15}	0.000580289	**0.00063437**	**0.000707201**	0.001545556	0.001225589	0.001411983	**0.000496184**	**0.000441549**	**0.000656427**	**0.000479392**	0.000871313
	(0.000362559)	(0.000385607)	(0.000543025)	(0.004456308)	(6.87867E-06)	(0.003855699)	(0.000386388)	(0.000345109)	(0.00043209)	(0.000307943)	(0.0004467)
f_{16}	-1.924472904	-1.924472224	-1.924472113	**-1.924473543**	-1.92447336	-1.924472505	-1.924473543	-1.924473528	-1.924473215	-1.924473016	-1.92447352
	(0.579460516)	(0.579460528)	(0.579460642)	(0.579460632)	(0.579460629)	(0.57946053)	(0.579460632)	(0.579460632)	(0.579460613)	(0.579460543)	(0.579460633)
f_{17}	3.78695256	3.786952596	3.786952688	**3.786952342**	**3.786952395**	3.78695256	3.786953207	**3.786952346**	3.786952426	3.786952433	3.786952348
	(2.374931129)	(2.374931132)	(2.374931083)	(2.374931169)	(2.374931161)	(2.374931106)	(2.374933216)	(2.374931167)	(2.374931153)	(2.374931141)	(2.374931167)
f_{18}	4.575001205	4.575011001	4.575028689	6.005761337	**4.574969199**	**4.575008958**	**4.57495705**	**4.574957813**	**4.574974777**	**4.574979692**	**4.57495876**
	(1.1176575)	(1.117655702)	(1.117663585)	(6.14484409)	(1.117659974)	(1.117664001)	(1.11765655)	(1.117656608)	(1.117661509)	(1.117662402)	(1.117656655)
f_{19}	-3.552074315	-3.552064298	-3.552065788	**-3.552084117**	-3.552080969	-3.552072265	**-3.552084001**	-3.552083898	-3.552079811	-3.552078513	-3.552083673
	(1.903087517)	(1.903082855)	(1.903081611)	(1.903090722)	(1.903089041)	(1.903085633)	(1.903090608)	(1.903090605)	(1.903088929)	(1.903089015)	(1.903090478)

Theoretical Motivation 1



1. A **search space** contains the <u>set of all possible solutions</u>.
2. An **objective function** determines the <u>quality of solution</u>.
3. A **(Mathematical idealized) metaheuristic** determines the <u>sampling order</u> (i.e. enumerates i.e. without replacement). It is a (approximate) permutation. What are we learning?
4. **Performance measure** *P* (*a*, *f*) depend only on y1, y2, y3
5. **<u>Aim</u> <u>find a solution with a near-optimal objective value using a Metaheuristic .</u>** ANY QUESTIONS BEFORE NEXT SLIDE?

Theoretical Motivation 2



$P(a, f) = P(a\,\boldsymbol{\sigma}, \boldsymbol{\sigma}^{-1} f)$ $P(A, F) = P(A\boldsymbol{\sigma}, \boldsymbol{\sigma}^{-1}F)$ (i.e. permute bins)

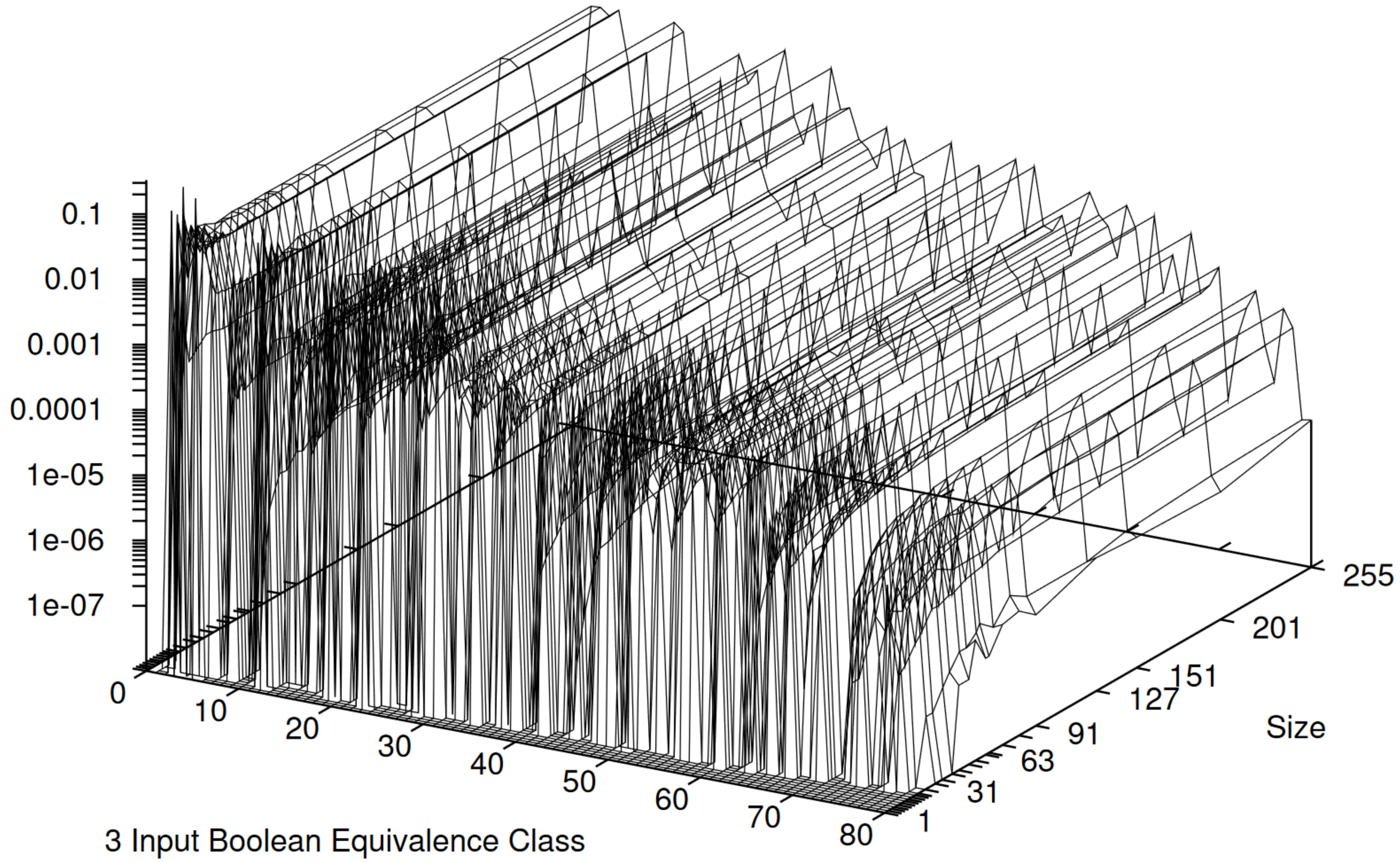
P is a **performance measure**, (based only on output values).

$\boldsymbol{\sigma}, \boldsymbol{\sigma}^{-1}$ are a permutation and inverse permutation.

A and F are probability distributions over algorithms and functions).

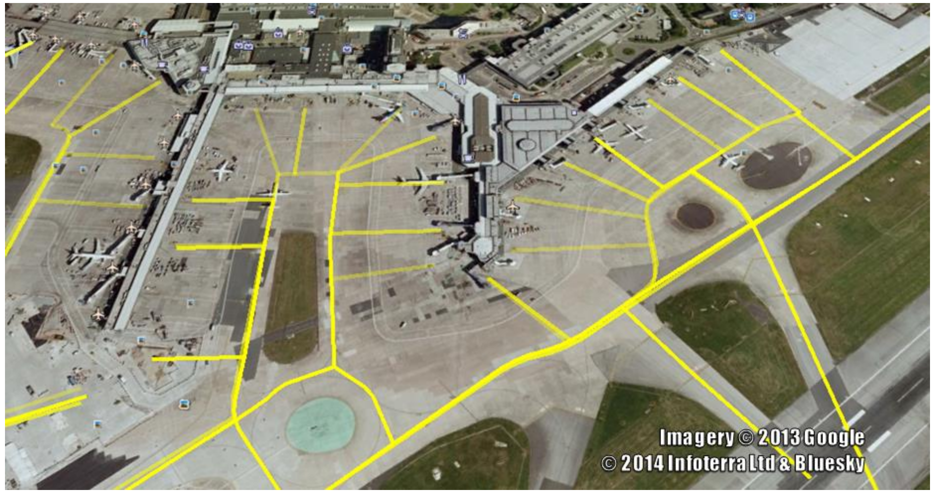
F is a problem class. **ASSUMPTIONS** **IMPLICATIONS**

1. Metaheuristic **a** applied to function $\boldsymbol{\sigma\sigma}^{-1}\boldsymbol{f}$ (that is $\boldsymbol{f}$)
2. Metaheuristic **a**$\boldsymbol{\sigma}$ applied to function $\boldsymbol{\sigma}^{-1}\boldsymbol{f}$ precisely identical.



0.1
0.01
0.001
0.0001
1e-05
1e-06
1e-07
0
10
20
30
40
50
60
70
80
3 Input Boolean Equivalence Class
1
31
63
91
127
151
201
255
Size

Ground Movements at Airport



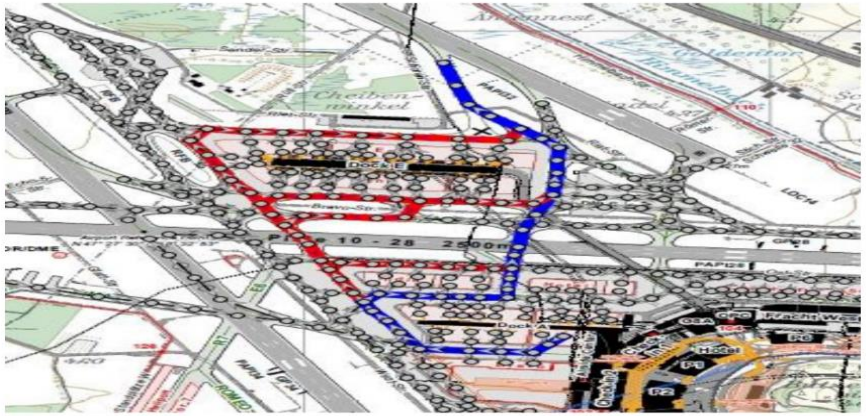
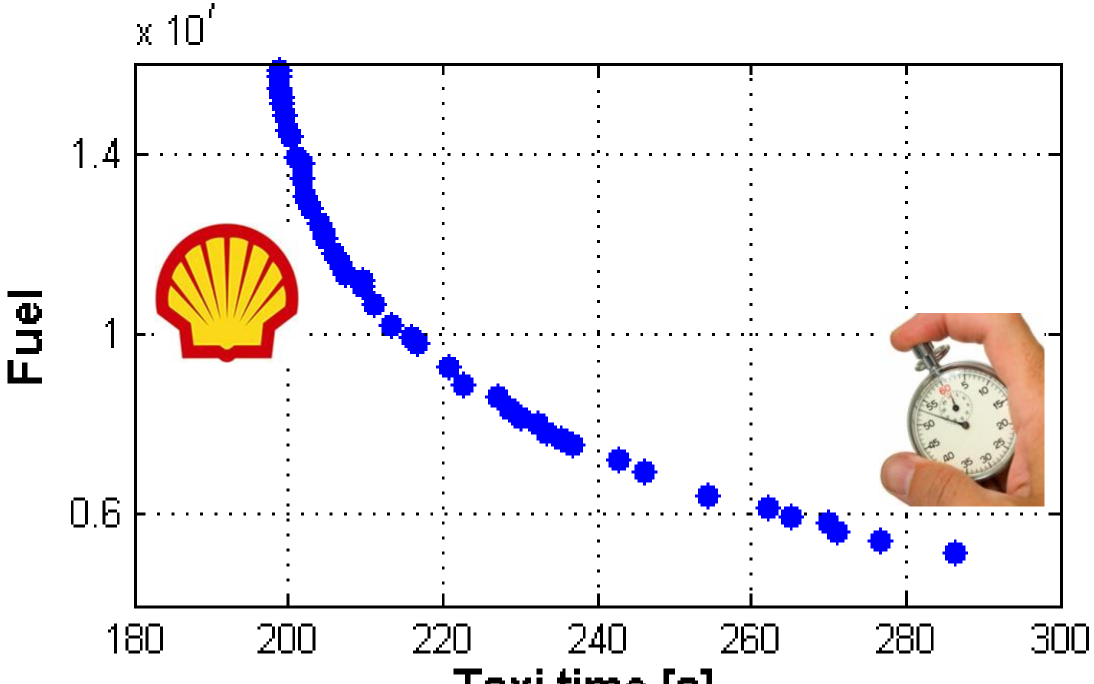


Fig. 2 Different routes from the exit of runway 14 to pier A

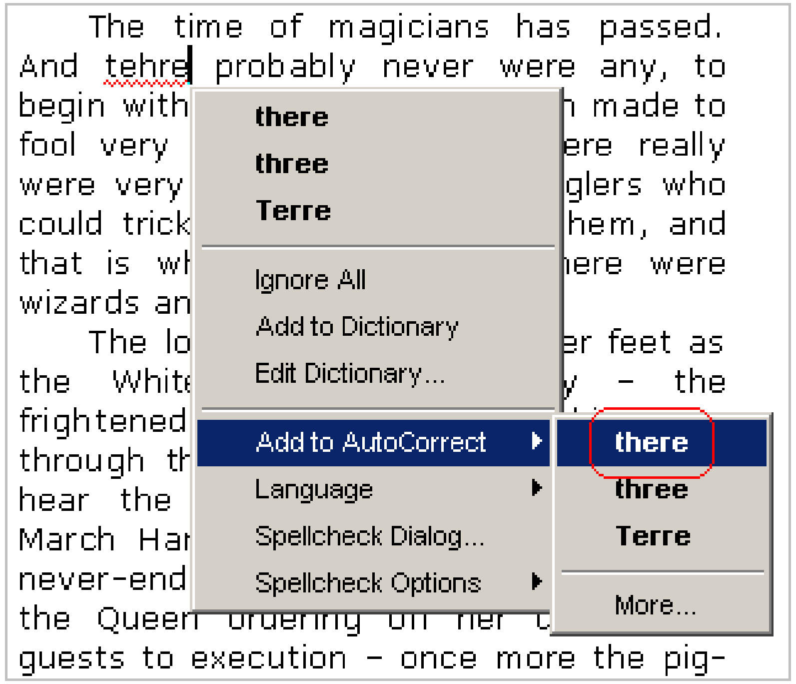


Automated Bug Fixing.

Table 1: Sets of single operators available to the GI. One member of a given set can be changed to another member of the same set.

Description	Operations
Numerical constants	Can increment by ± 1
Arithmetic operators	$+, -, *, /, //, \%, **$
Arithmetic assignments	$+=, -=, *=, /=,$
Relational operators	$<, >, <=, >=, ==, !=,$ *is*, *is not*, *not*
Logical operators	*and*, *or*
Logical constants	*True*, *False*

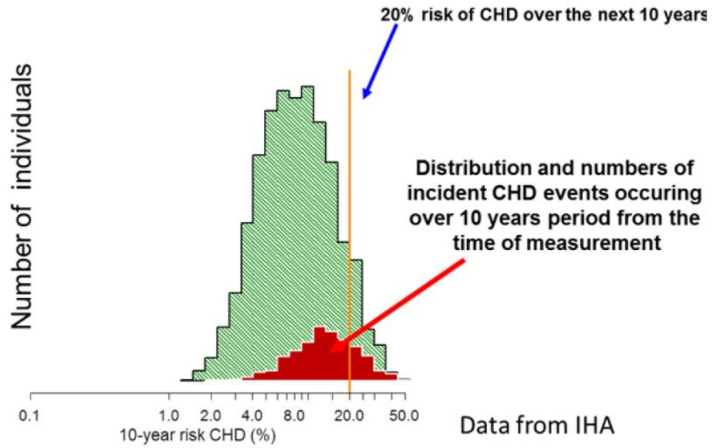
- Machine learning
- detect bug location
- suggest bug fix



Heart disease predictor.

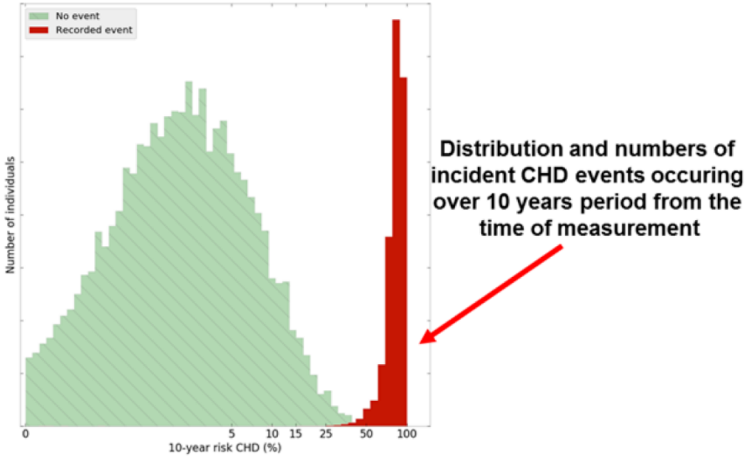
```
((kwargs['kvk_10'] * einst['FAMILYMI_Y']) +
        ((kwargs['kvk_11'] * einst['PREVSMOKER']) +
        (((kwargs['kvk_12'] * (einst['CHOL'] -
kwargs['kvk_13'])) * einst['SMOKER']) +
        ((kwargs['kvk_14'] * einst['DM2']) -
        ((kwargs['kvk_15'] *
einst['SPORTSCURRENT']) -
        (kwargs['kvk_16'] * ((einst['HDL'] -
einst['DM2']) / kwargs['kvk_17'])))))))))
```

Fig 1.4.1. Distribution of the %risk of suffering CHD event over the next 10 years in general population of men aged between 40 and 70 years

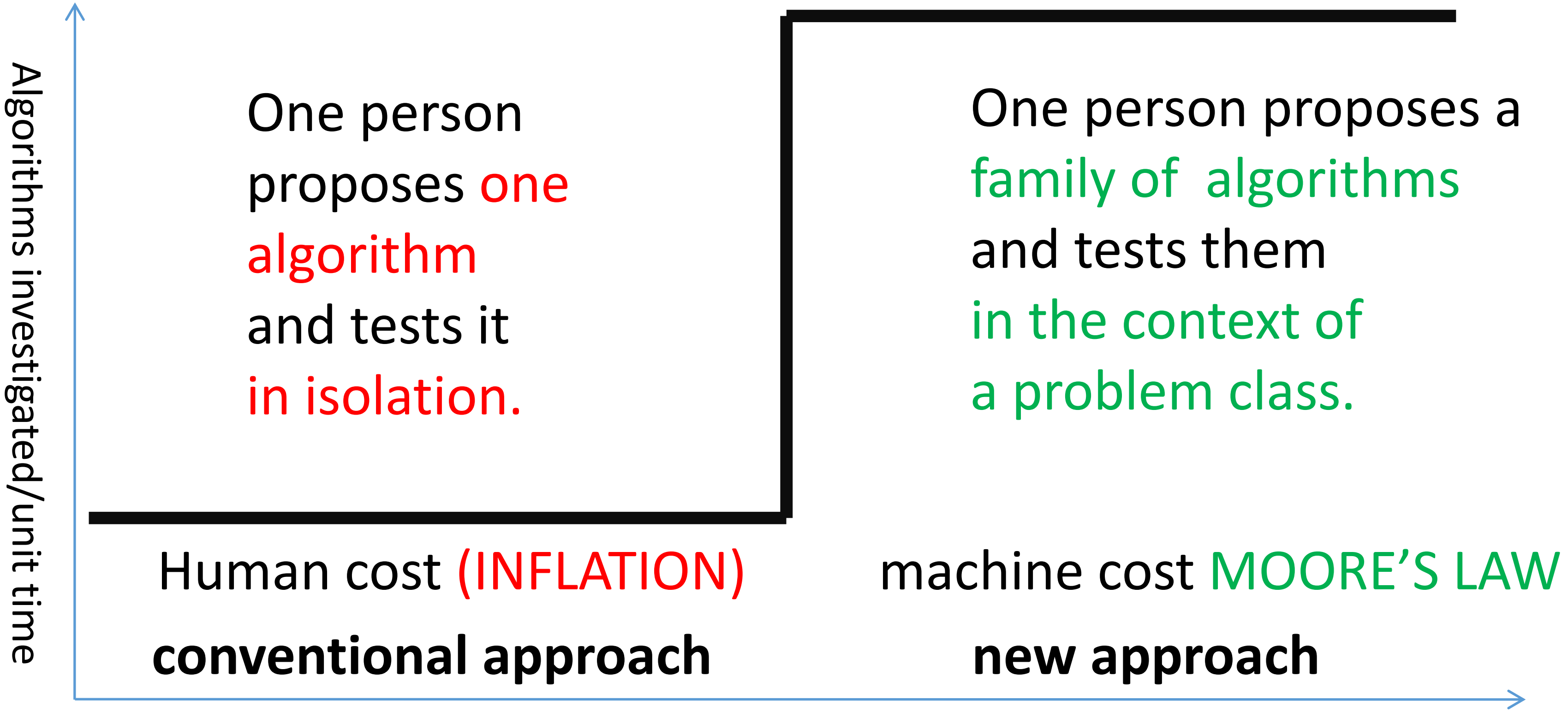


Data from IHA

Fig 1.4.2. Distribution of the %risk of suffering CHD event over the next 10 years in general population of men aged between 40 and 70 years



A Paradigm Shift?



- Previously **one** person proposes **one** algorithm
- Now **one** person proposes **a set of** algorithms
- Analogous to "industrial revolution" from hand made to machine made. Automatic Design.

Thank you. Any questions.

- Applications
  - *Airport ground movements.*
  - *Software engineering*
  - *Medicine – heart disease indicator*

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http://gpbib.cs.ucl.ac.uk/gp-html/index.html (40th / 10,000. 2nd largest AI BIB)
https://scholar.google.co.uk/citations?user=iZIjJ80AAAAJ&hl=en
https://gow.epsrc.ukri.org/NGBOViewPerson.aspx?PersonId=-485755

Summary