Parton showers beyond leading logarithmic accuracy

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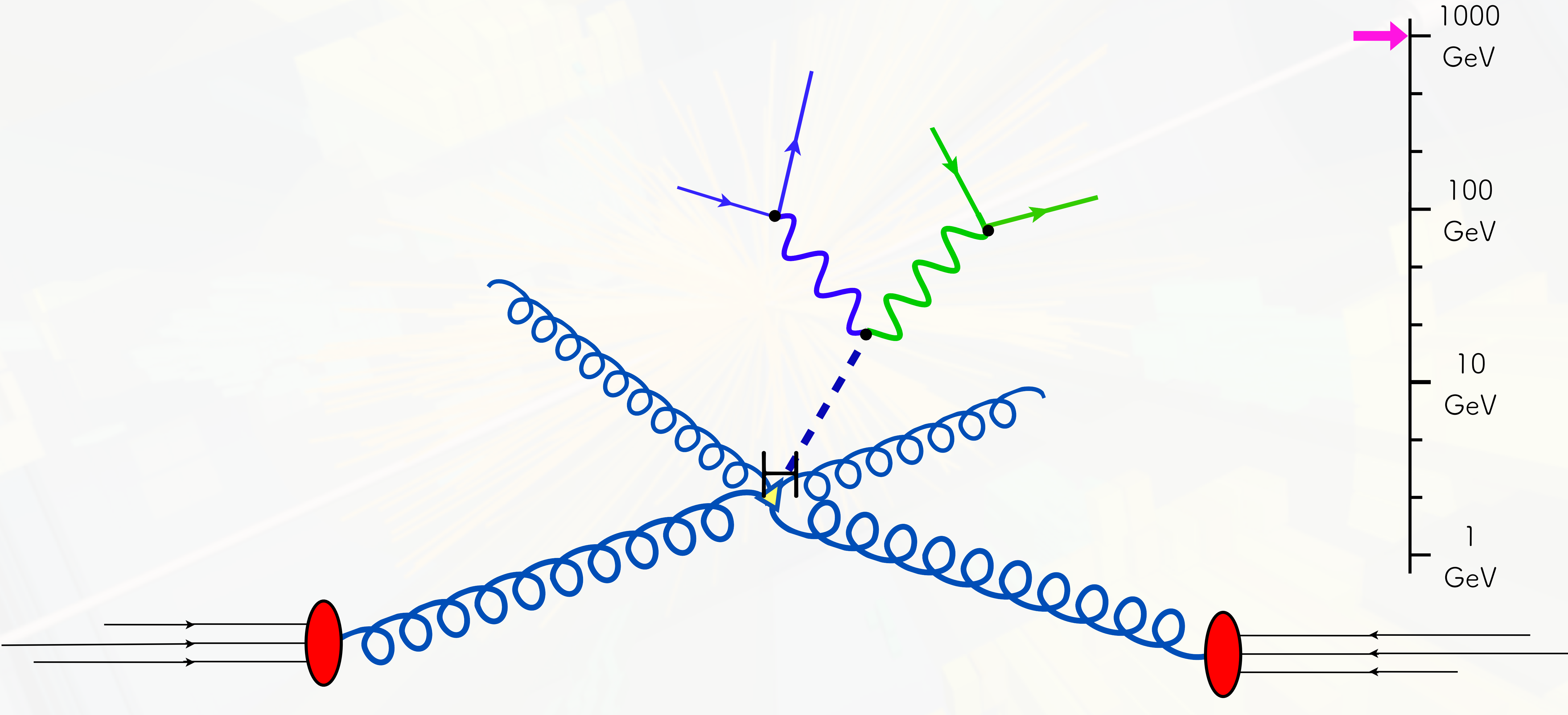
Mrinal Dasgupta, Frédéric Dreyer, Pier Monni, Gavin Salam, Gregory Soyez : arXiv:2002.11114

European Research Council

relevance

Parton Shower Event Generators

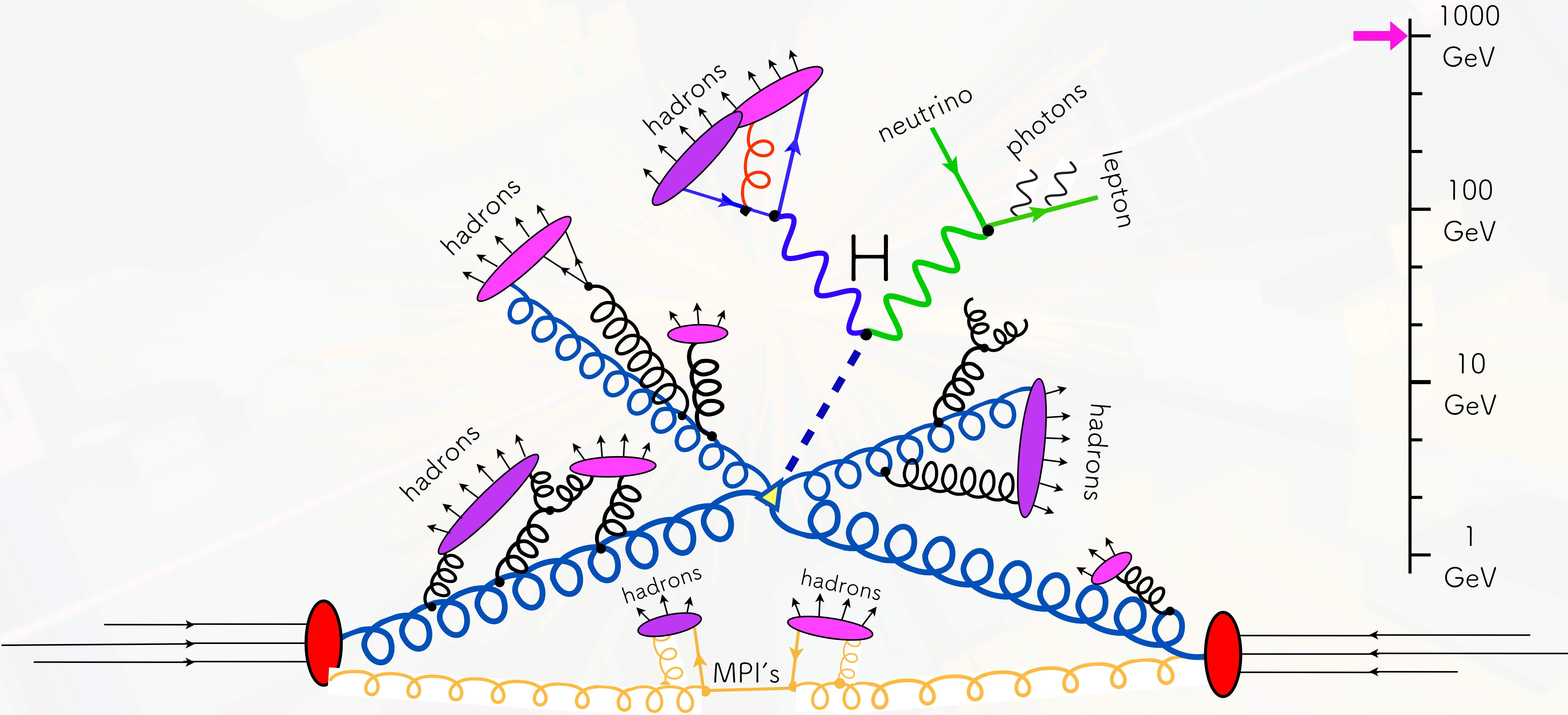
- PS MCs take hard scattering events generated according to fixed order pQCD characterised by mom. transfers in the 100's to 1000's of GeV



Parton Shower Event Generators

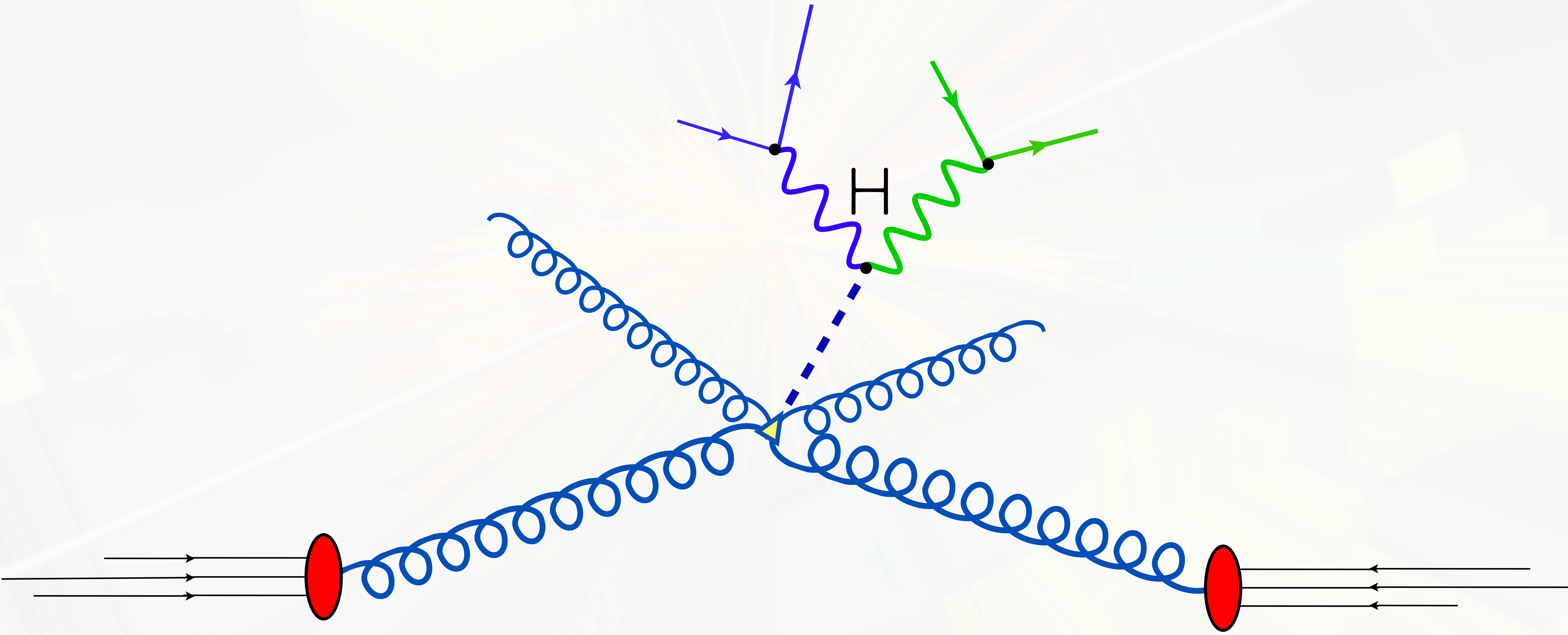
- they dress them with increasingly soft, real & virtual, resummed radiative corr^{n}s, accounting for the structure of events at ever smaller scales

down to the scale where hadrons form, O(1 GeV)

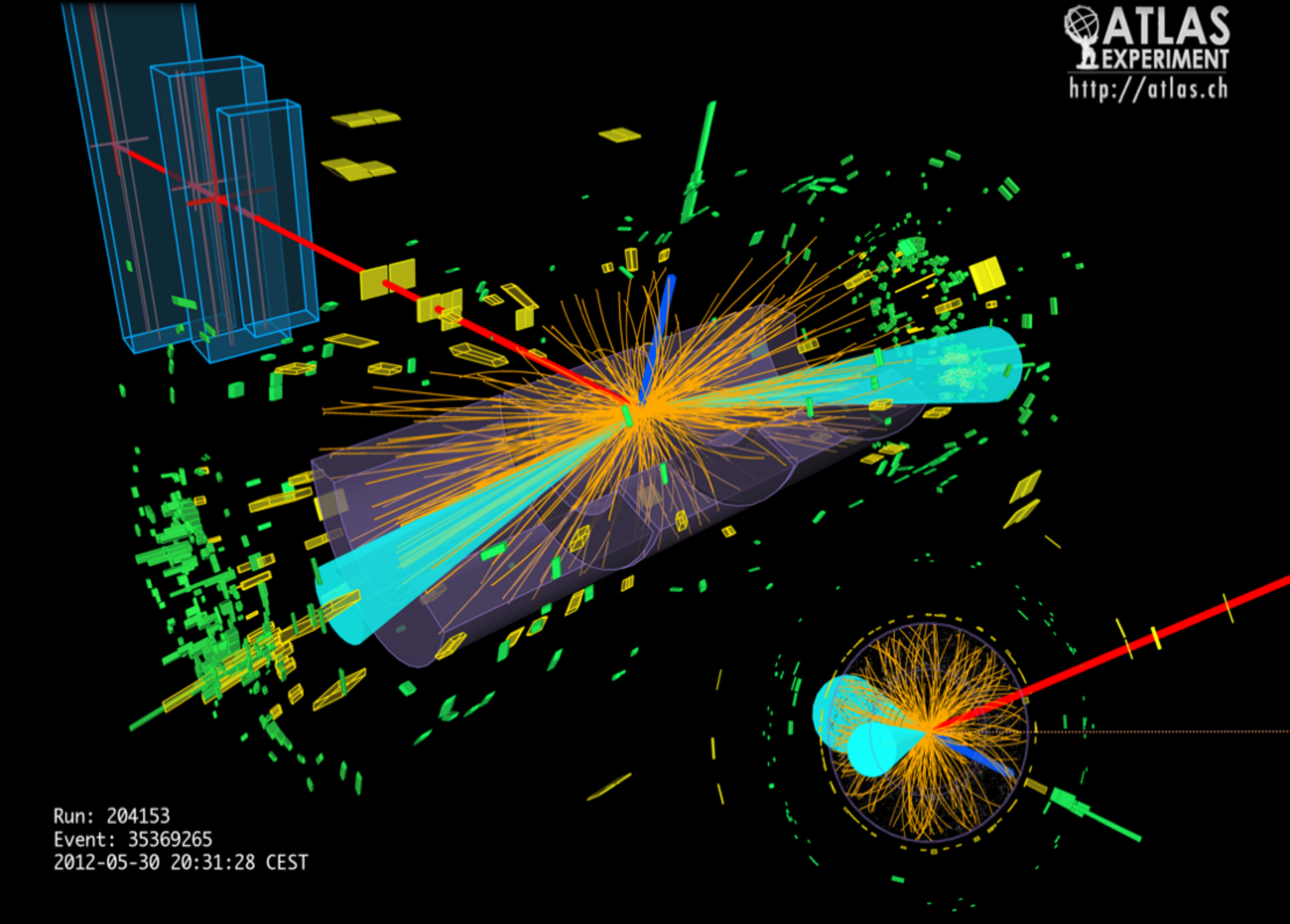


Parton Shower Event Generators

- They take us from this level of theoretical description



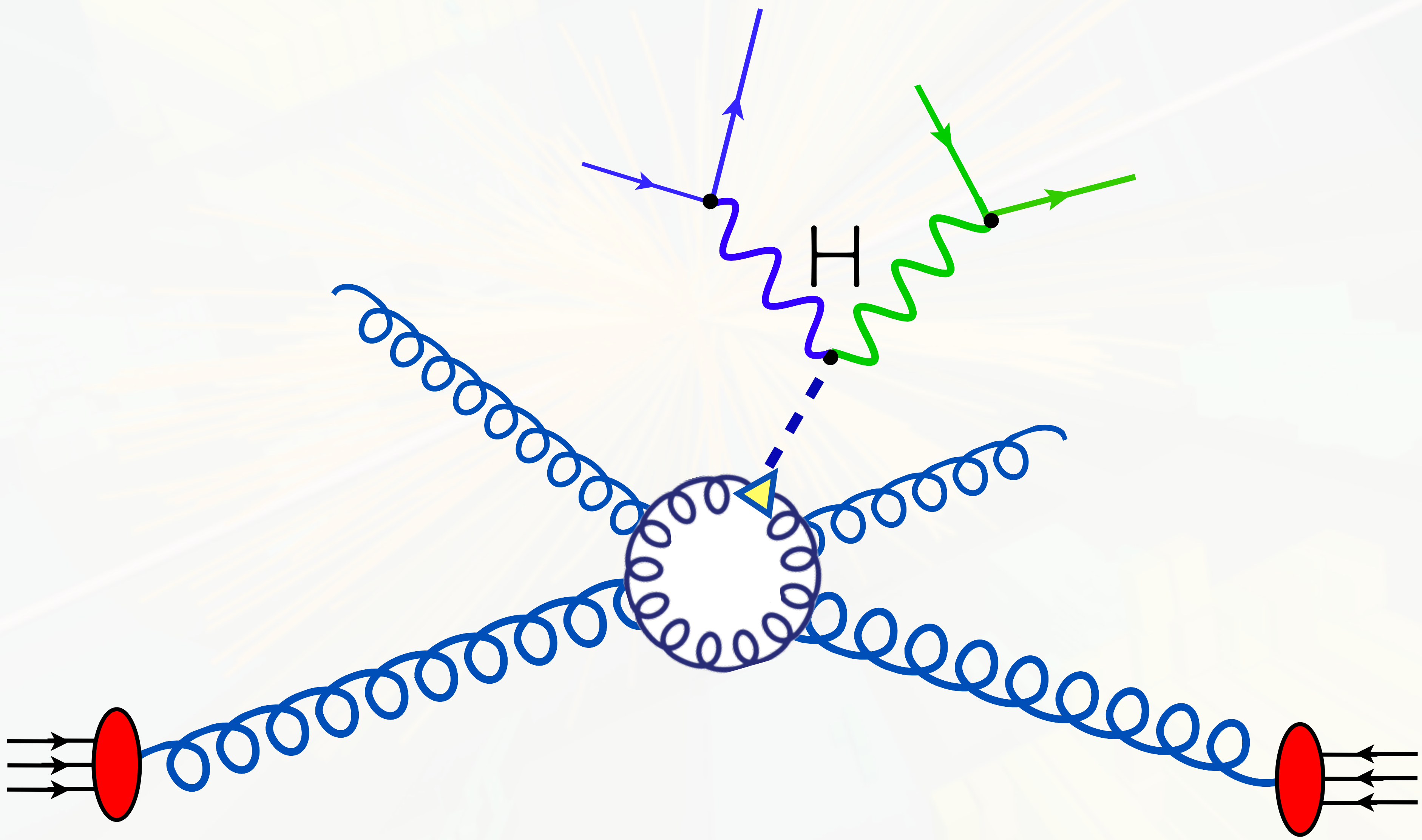
to this ...



ATLAS EXPERIMENT
http://atlas.ch
Run: 204153
Event: 35369265
2012-05-30 20:31:28 CEST

NLO + Parton Shower Event Generators

- Incisive new theory c. 2004 — AMC@NLO and POWHEG — showed how to fuse PSMC consistently with NLO perturbation theory, in generality and in practice

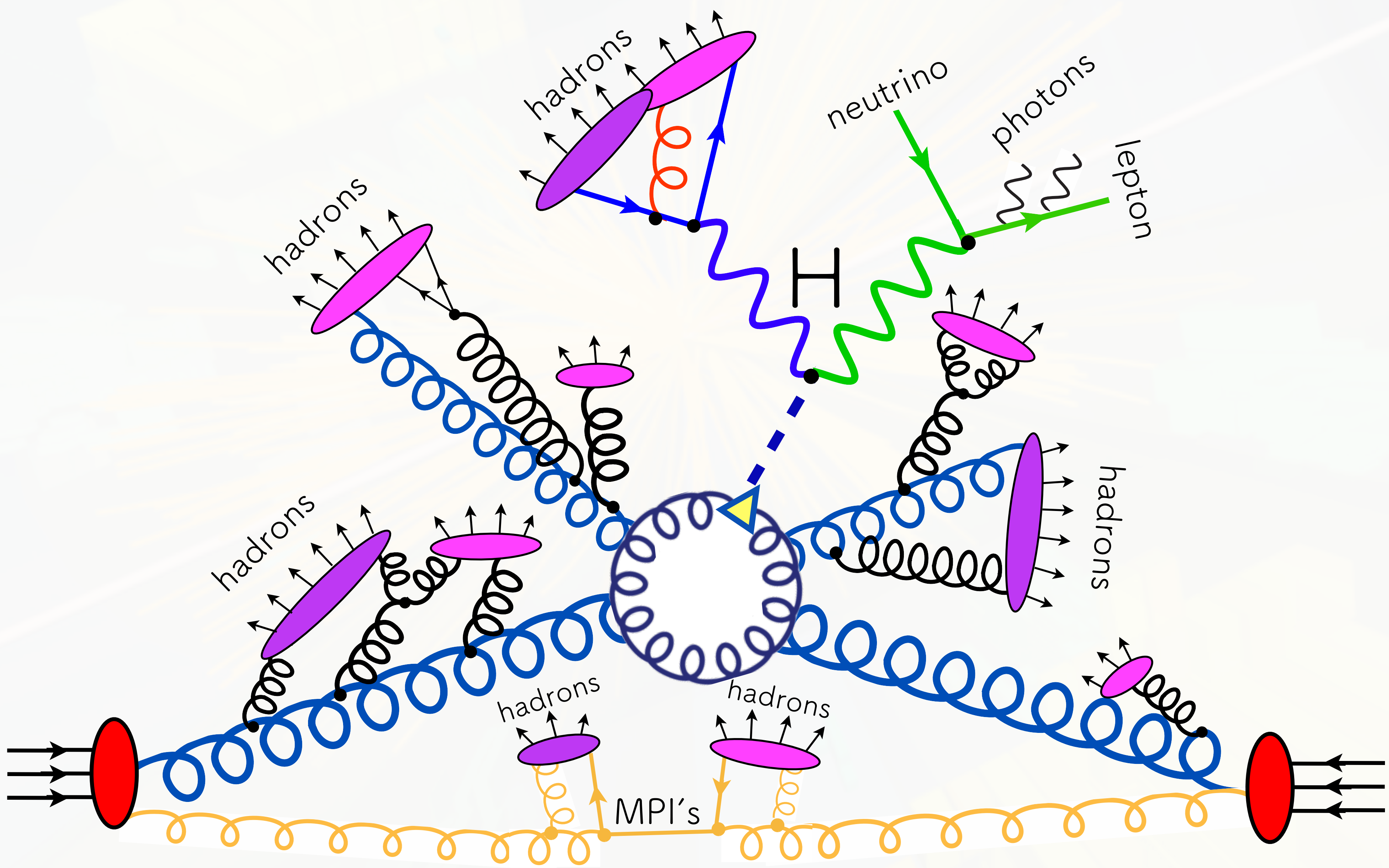


MC@NLO: Frixione, Webber

POWHEG: Alioli, Nason, Oleari, Re

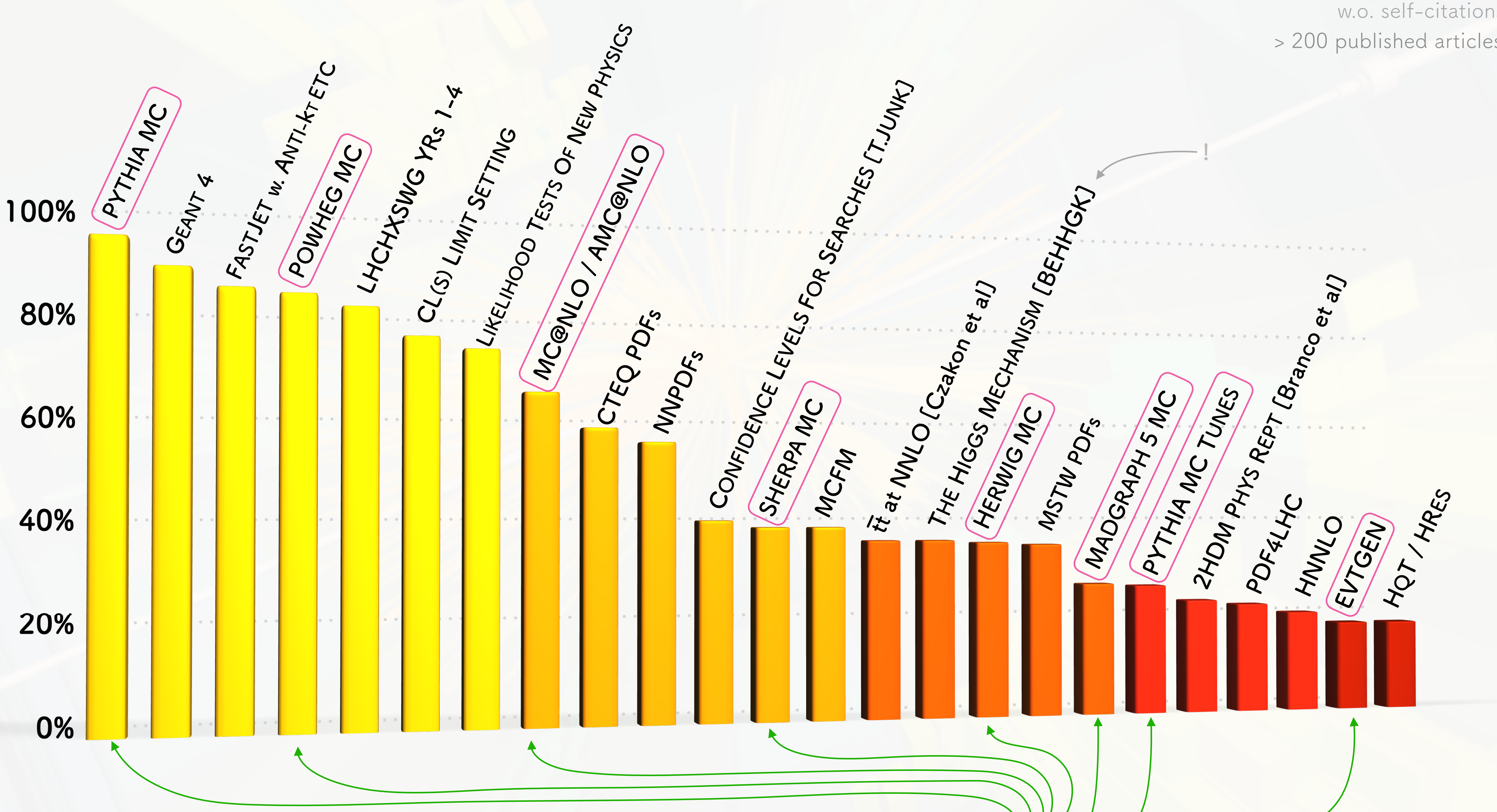
NLO + Parton Shower Event Generators

- NLO+PS codes for basically all SM processes of interest can be freely obtained from a number of teams : HERWIG, AMC@NLO, SHERPA, POWHEG-BOX



Does it matter, e.g. for Higgs?

- % of ATLAS+CMS+LHCb papers citing an article/group in Jan '14 → Oct '19



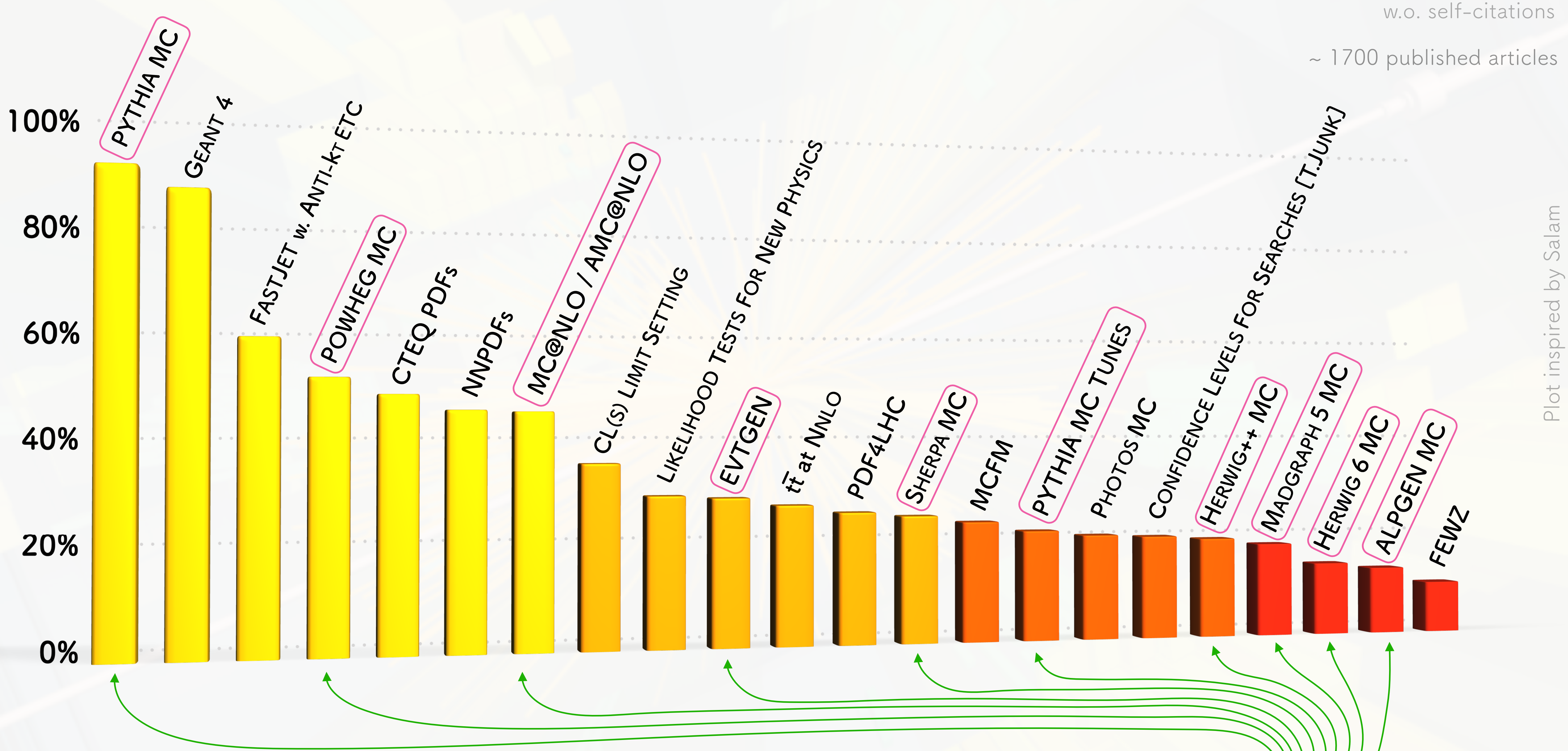
- PS / NLO+PS MC ubiquitous in Higgs analysis
- PYTHIA features prominently in 93% of papers, POWHEG 83%, AMC@NLO 66%

Does it matter for anything else?

- % of ATLAS+CMS+LHCb papers citing some article/group in Jan '14 → May '20

w.o. self-citations

~ 1700 published articles

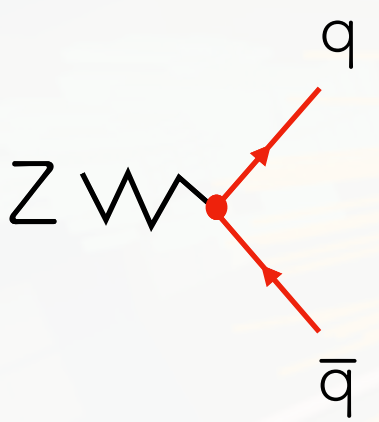


- PS MC is a central, everyday, part of the LHC physics programme
- In simulating arbitrary numbers of real/virtual, soft/collinear, emissions PS MCs are `the ultimate' resummation tools : they give you pred^{n}s for any observable

dipole parton shower in 2 mins

dipole shower in 2 mins

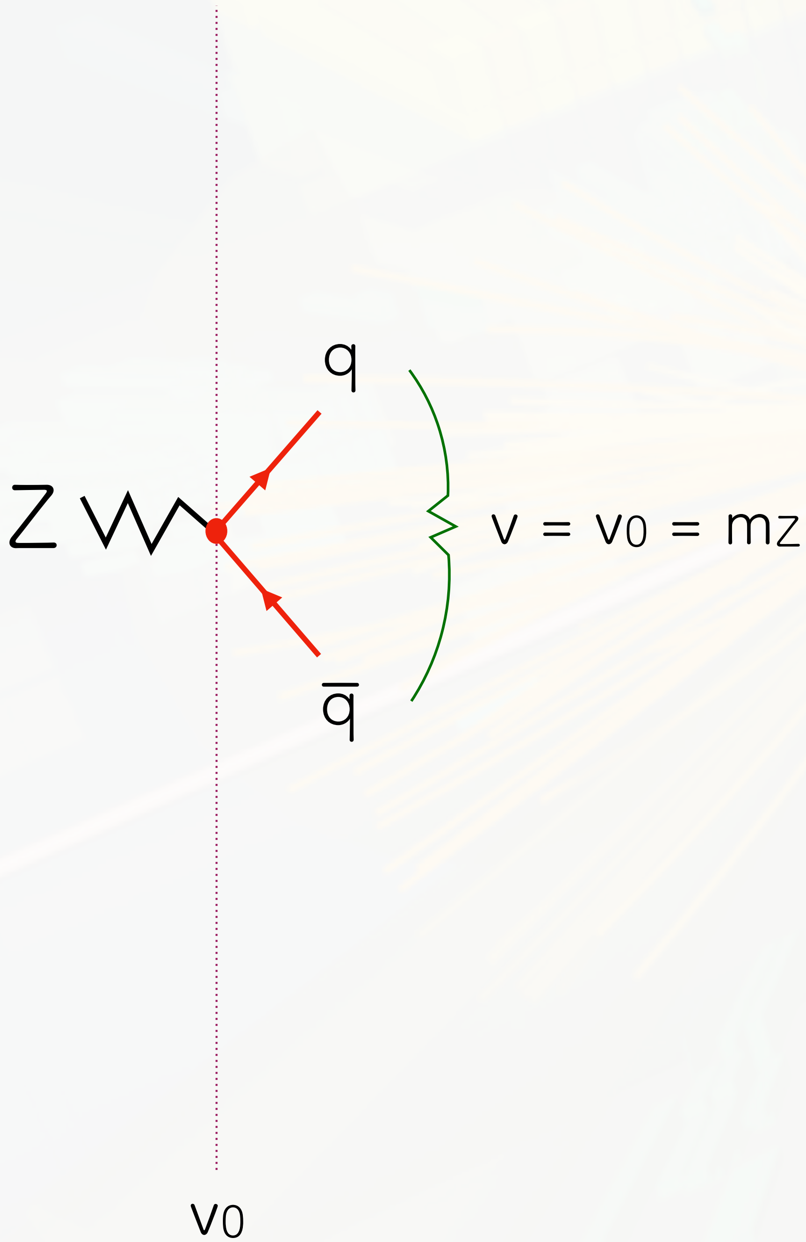
- Events are viewed throughout as a collection of colour-anticolour dipole ends, starting already at the level of the hard scattering



- gluon has two dipole ends, colour & anti-colour
- quark has one colour dipole end
- antiquark has one anti-colour dipole end

dipole shower in 2 mins

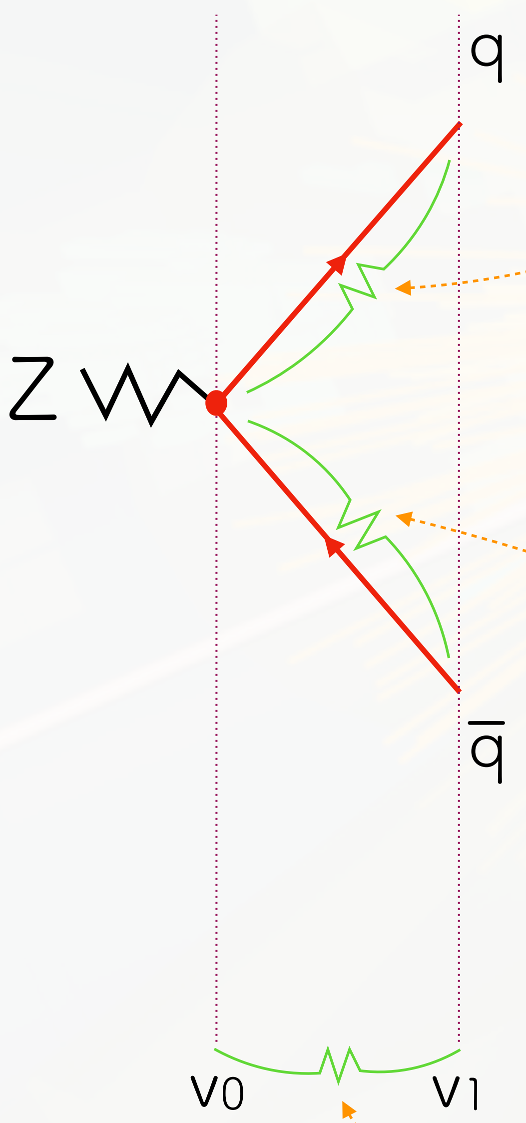
- Events are viewed at resolution scale v : typically the min $p_\perp$ sep^n of any two partons
- Zooming-out from large v to small v more partons get resolved with smaller $p_\perp$ sep^{n}s



- Changing resolution like this is referred to as evolution in shower time [i.e. v]

dipole shower in 2 mins

- Probability to evolve [zoom-out] from resolution scale v_0 to some smaller scale v_1 without resolving anything new along the way is called the Sudakov form factor
- It's the product of probabilities for no resolvable emission from each dipole end



P[q dipole end doesn't emit in $v_0 \rightarrow v_1$]

$$\Delta_q(v_0, v_1) = \exp\left[-\int_{v_1}^{v_0} d\mathcal{P}_{q[\bar{q}]\to qg[\bar{q}]}\right]$$

P[q dipole end doesn't emit in $v_0 \rightarrow v_1$]

$$\Delta_{\bar{q}}(v_0, v_1) = \exp\left[-\int_{v_1}^{v_0} d\mathcal{P}_{\bar{q}[q]\to \bar{q}g[q]}\right]$$

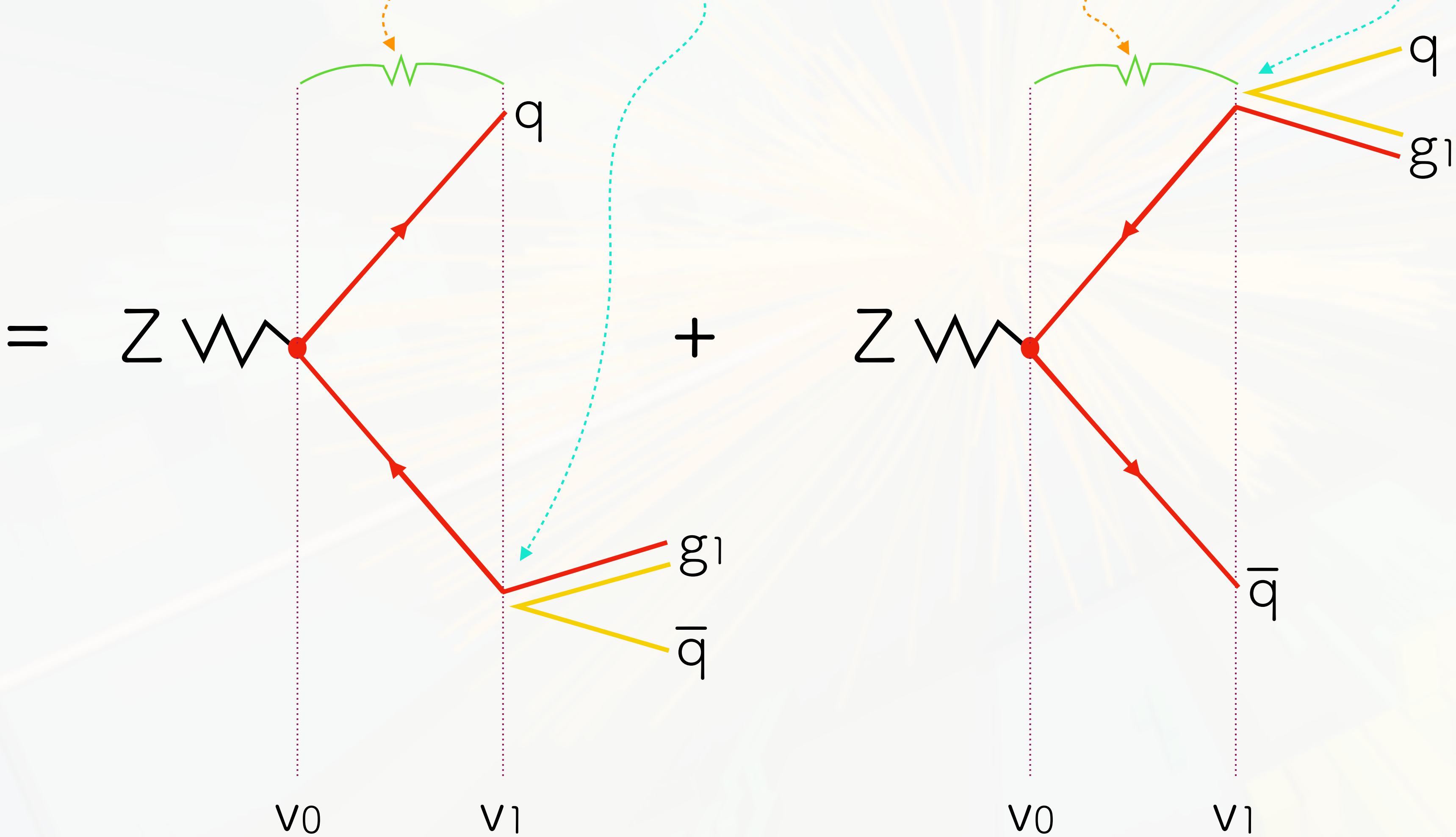
P[nothing in event emits in $v_0 \rightarrow v_1$]

$$\Delta(v_0, v_1) = \prod_{\substack{\text{dipole} \\ \text{ends } \tilde{i}}} \Delta_{\tilde{i}}(v_0, v_1)$$

dipole shower in 2 mins

- The first emission is distributed according to the probability neither side of the dipole emits and then either the colour end or the anti-colour end emits

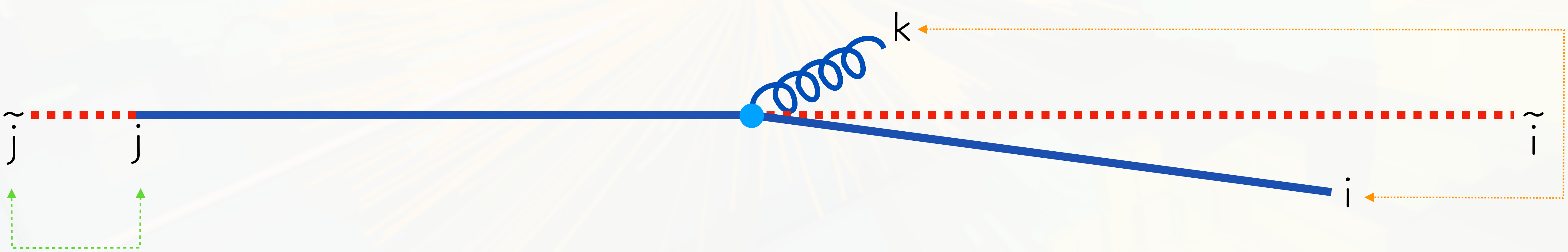
$$\Delta\left(v_0, v_1\right)\, d\mathcal{P}_{\bar{q}[q]\to\bar{q}g[q]} + \Delta\left(v_0, v_1\right)\, d\mathcal{P}_{q[\bar{q}]\to qg[\bar{q}]}$$



- Given an emission from a dipole $\tilde{i}\,\tilde{j}$, i.e. given a phase space point, the probability it comes the $\tilde{i}$ side is $d\mathcal{P}_{\tilde{i}[\tilde{j}]\to ig[j]}$, and analogously for the $\tilde{j}$ side $d\mathcal{P}_{\tilde{j}[\tilde{i}]\to jg[i]}$

dipole shower in 2 mins

- The total momentum in the event is conserved before and after the emission
- Other particles need to recoil to balance emission k
- E.g. in PYTHIA & DIRE emitter, i, takes the transverse recoil of k in the $\tilde{i}\,\tilde{j}$ C.O.M frame

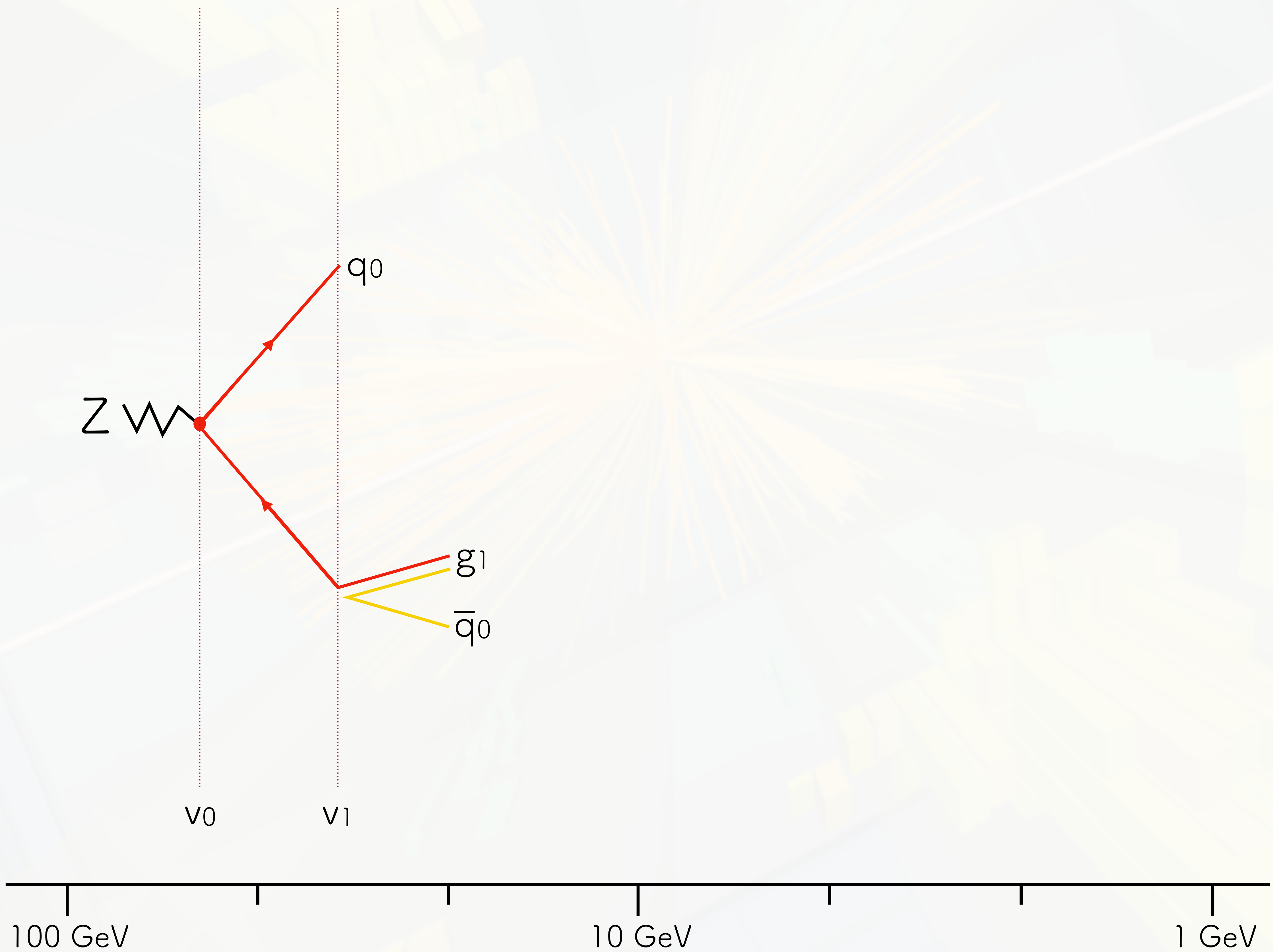


- Residual dipole longitudinal momentum imbalance absorbed by rescaling $\tilde{j} \rightarrow j$

[spectator]

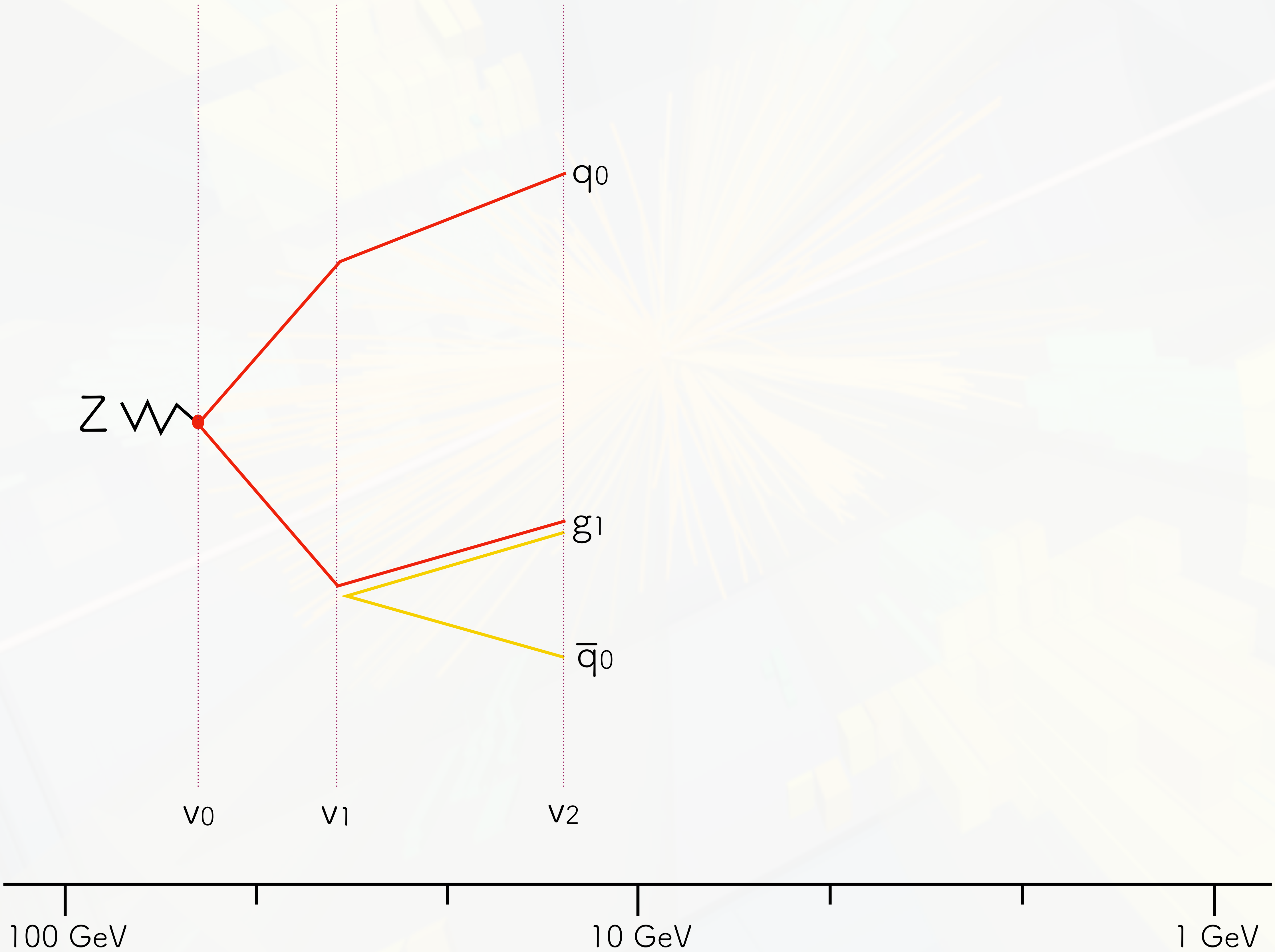
dipole shower in 2 mins

- Repeat the same exercise using the two new dipoles …



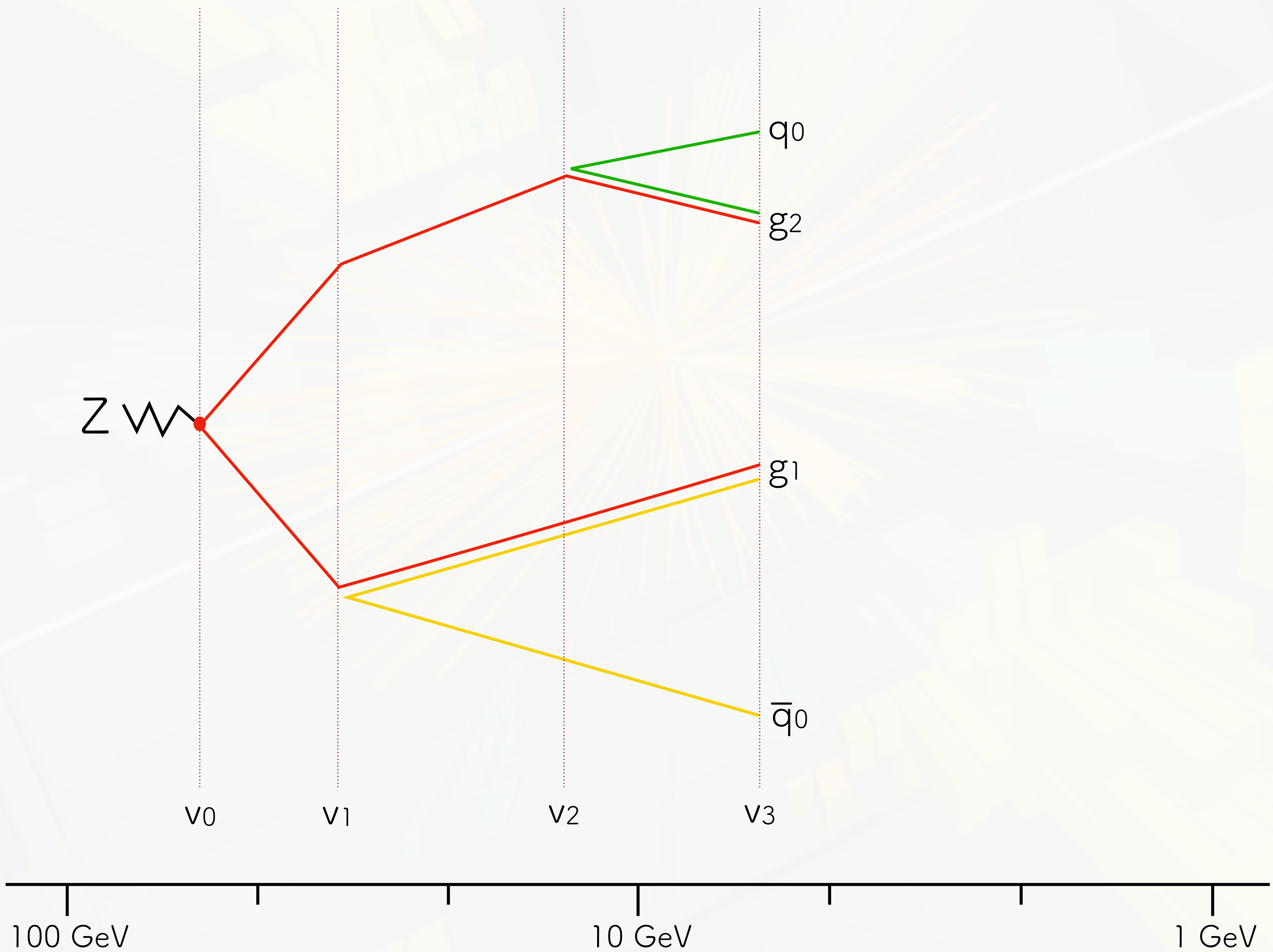
dipole shower in 2 mins

- Evolve without resolving anything ...



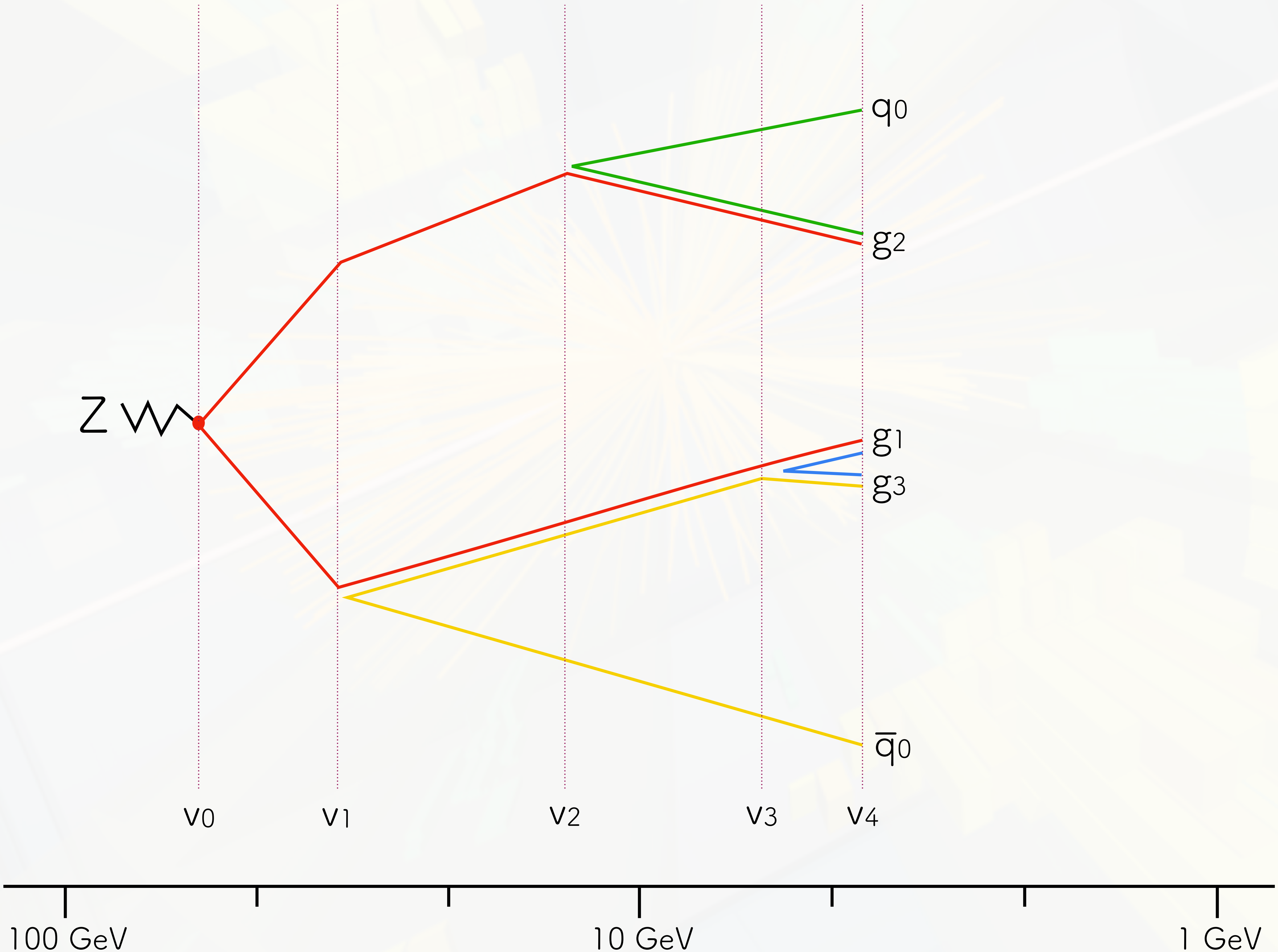
dipole shower in 2 mins

- Branch. Construct post-branching kinematics.



dipole shower in 2 mins

- And repeat ...



dipole shower in 2 mins

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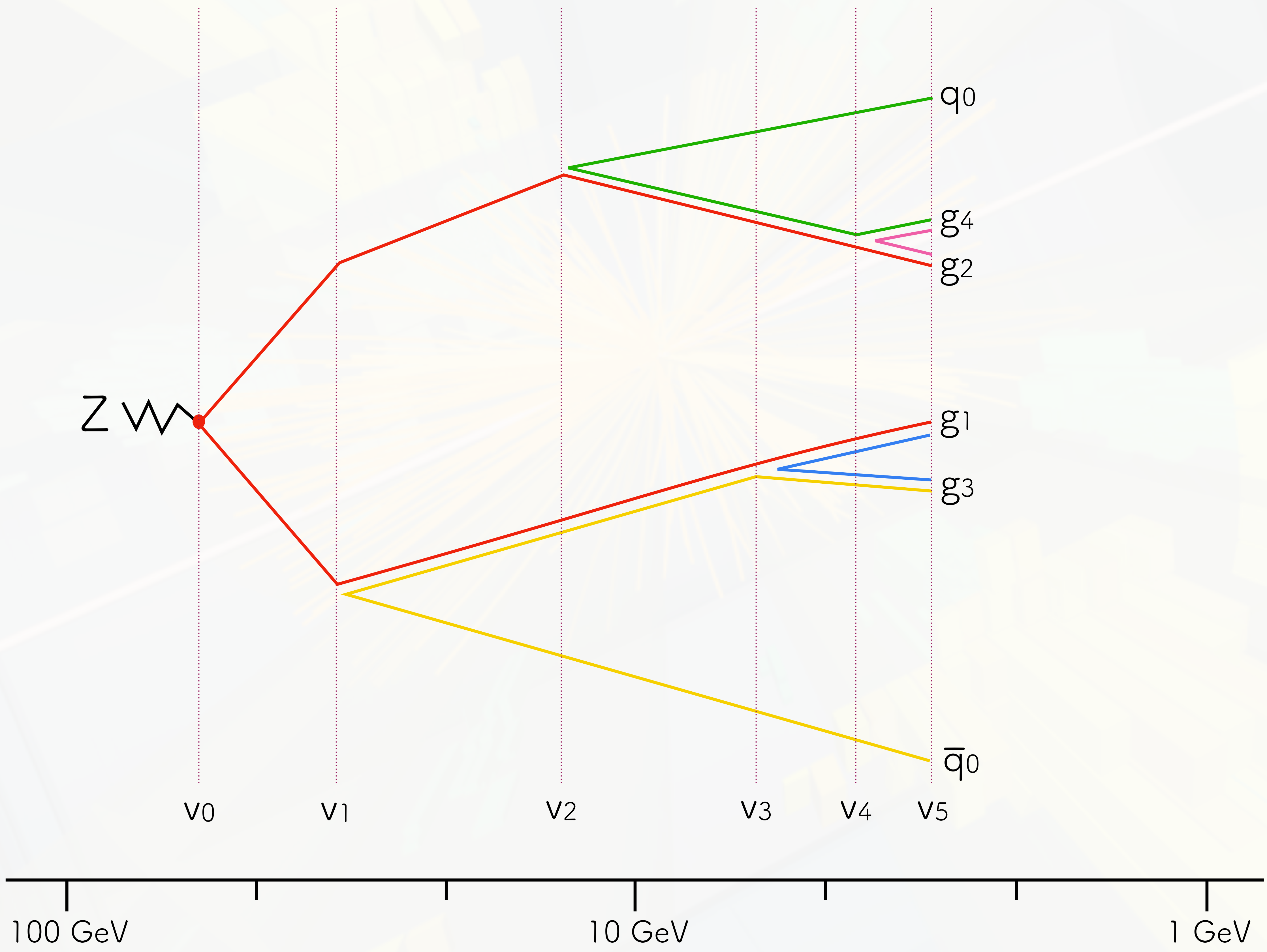

dipole shower in 2 mins

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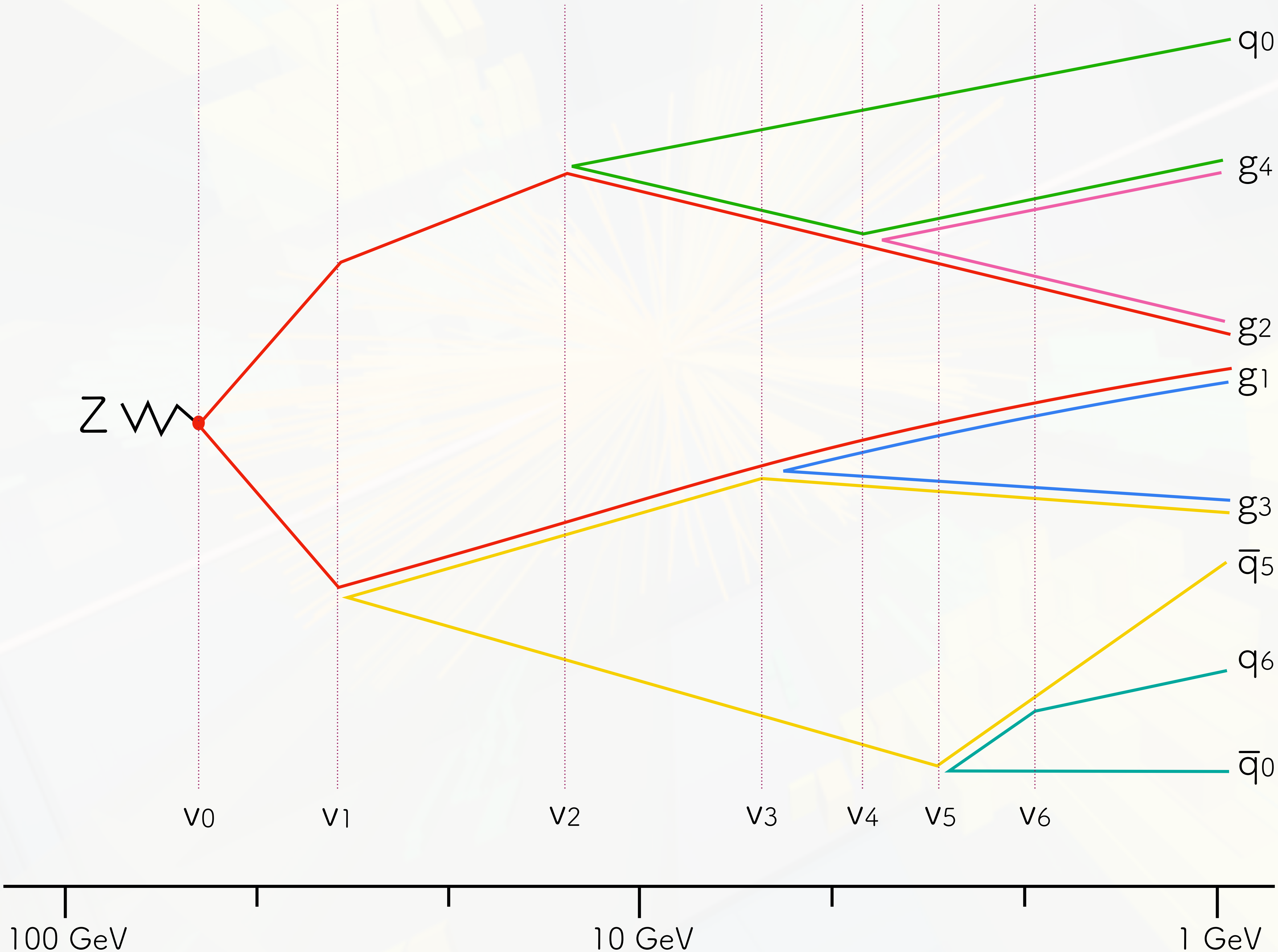
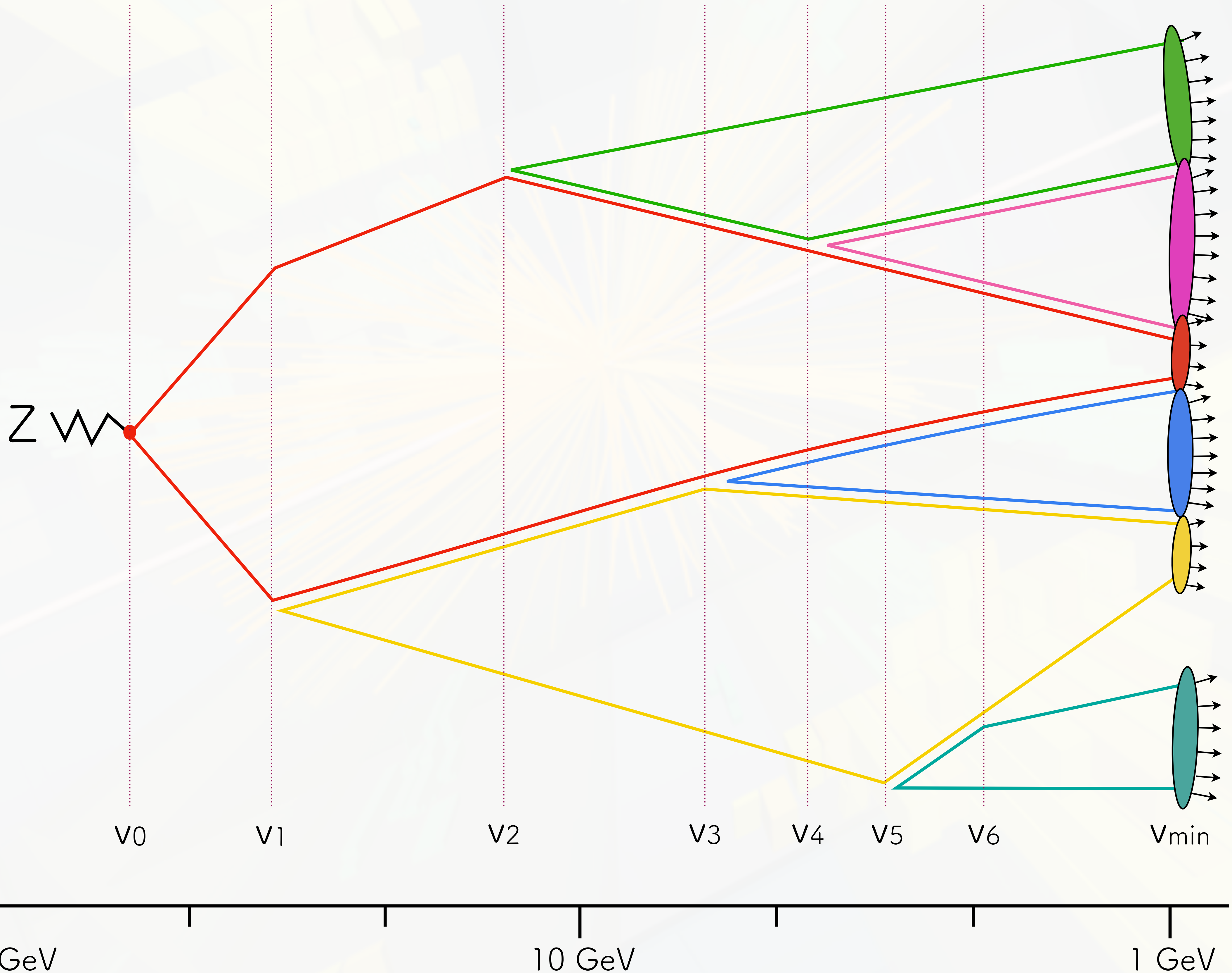


dipole shower in 2 mins

- Then hadronize for $v < v_{min}$



parton shower accuracy

parton shower accuracy

- PS MC accuracy not clearly defined; often conflated with ability to describe data
- PS MCs use ab initio QCD to simulate multiple-emissions across disparate scales
- This work:
  a) Define PS MC accuracy as ability to reproduce pQCD results for multiple-emissions across disparate scales ...
     i. singular structure of multiple parton MEs
     ii. logarithmic resummation results
  b) Establish general design principles for PS MCs to reach NLL accuracy
  c) Demonstrate with full-fledged concrete examples

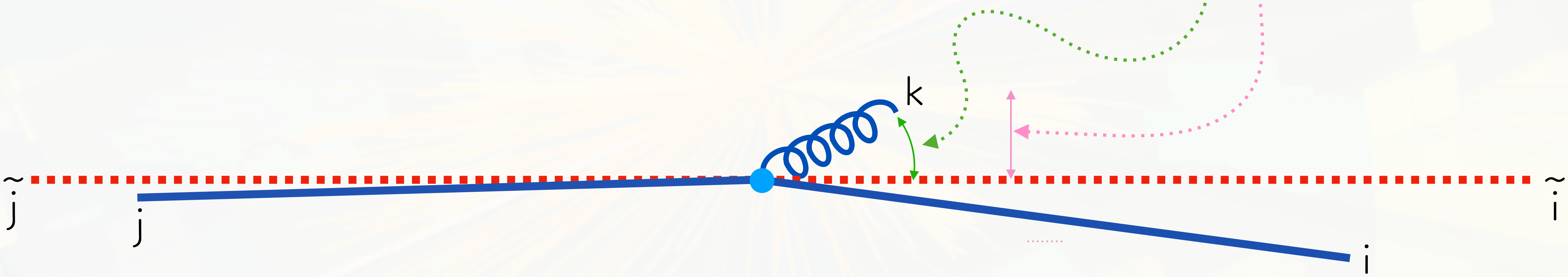
leading and next-to-leading log

single emission

- Given a FS colour dipole end, i, connected to an anti-colour dipole end, j

  the probability to emit a soft gluon into phase space element at $p_\perp, \eta_{\text{dip}}, \phi$ is

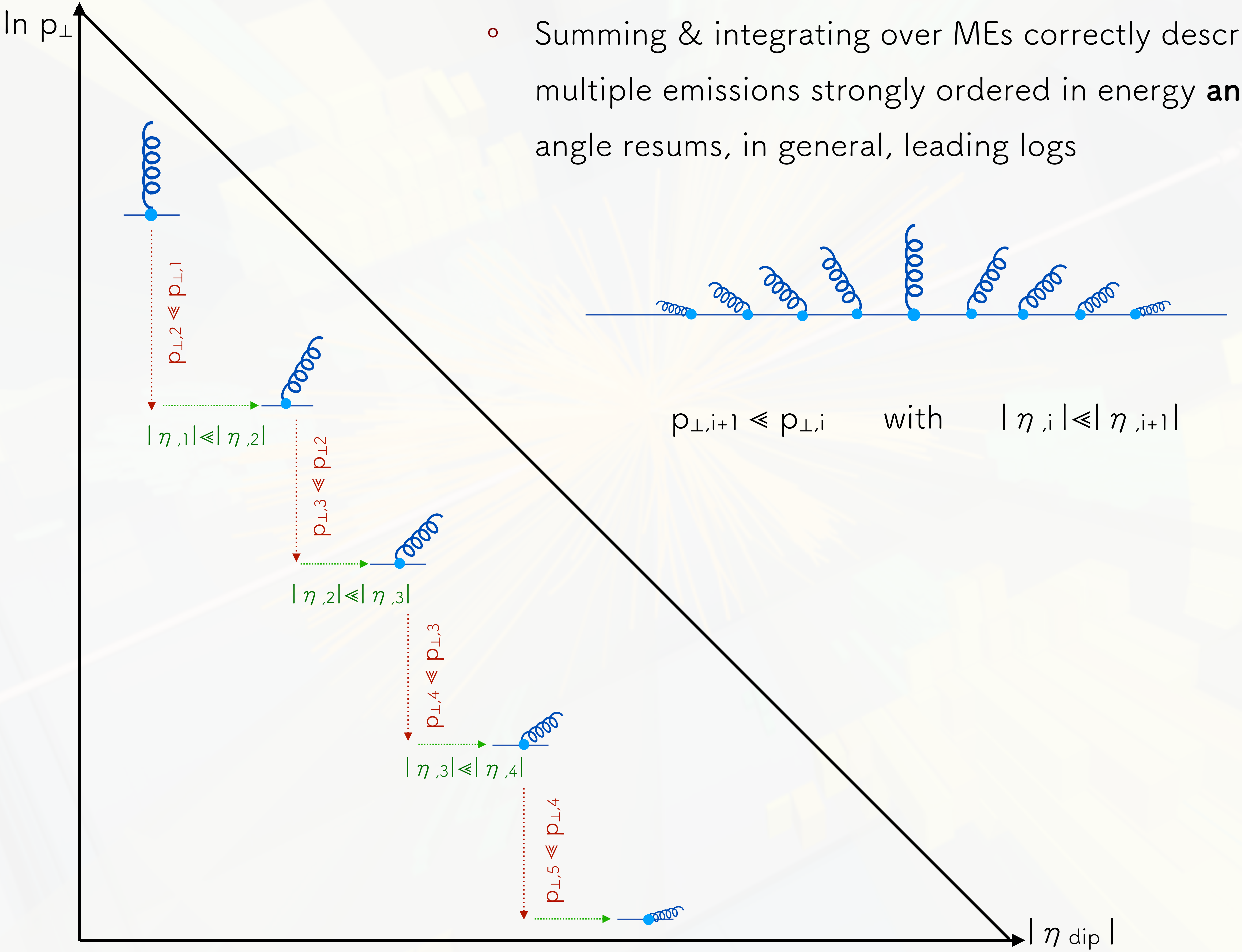
$$dP_{\tilde{i}\tilde{j}\to ikj} = \frac{\alpha_s C}{2\pi}\, dp_\perp^2\, d\eta_{\text{dip}}\, d\phi\, \frac{\tilde{p}_i.\tilde{p}_j}{\tilde{p}_i.p_k\, \tilde{p}_j.p_k} = \frac{2\alpha_s C}{\pi}\, d\ln p_\perp\, d\eta_{\text{dip}}\, d\phi$$



- Emission specified by three phase space vars, e.g. $p_\perp, \eta_{\text{dip}}$, two of which are logarithmic
- Single log: integrate over either $\ln p_\perp$ / η_{dip} [more generally over a line in $\ln p_\perp - \eta_{\text{dip}}$]
- Double log: integrate over both $\ln p_\perp$ & η_{dip} [more generally over an area in $\ln p_\perp - \eta_{\text{dip}}$]
- Can reparametrise, e.g. $p_\perp \to E_k \sim \frac{1}{2} p_\perp e^{|\eta|}$, but story above is the same

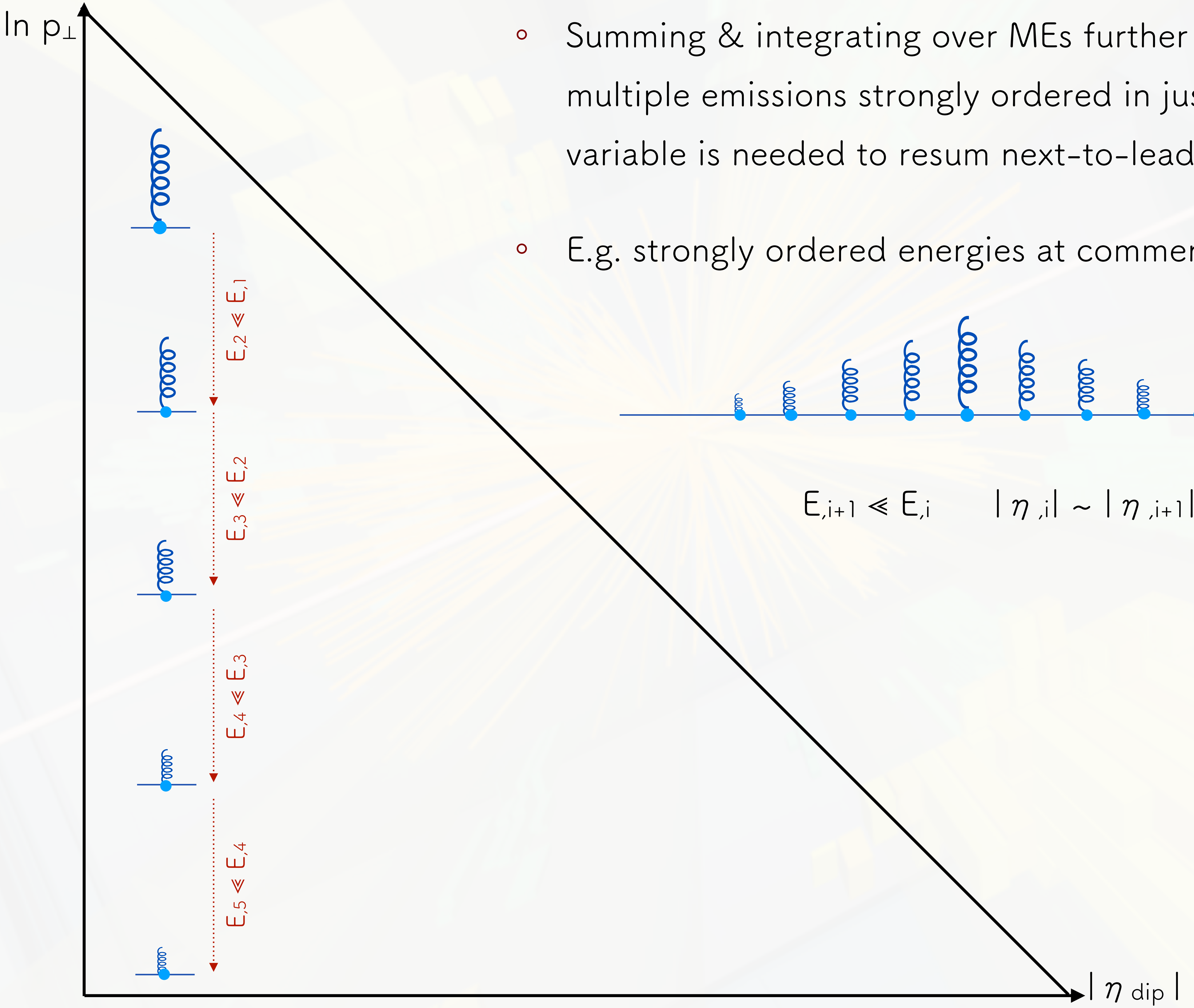
leading logs : strongly ordered emissions in both logarithmic variables

- Summing & integrating over MEs correctly describing multiple emissions strongly ordered in energy **and** angle resums, in general, leading logs



$p_{\perp,i+1} \ll p_{\perp,i}$ with $|\eta_{,i}| \ll |\eta_{,i+1}|$

leading logs : strongly ordered emissions in one logarithmic variable

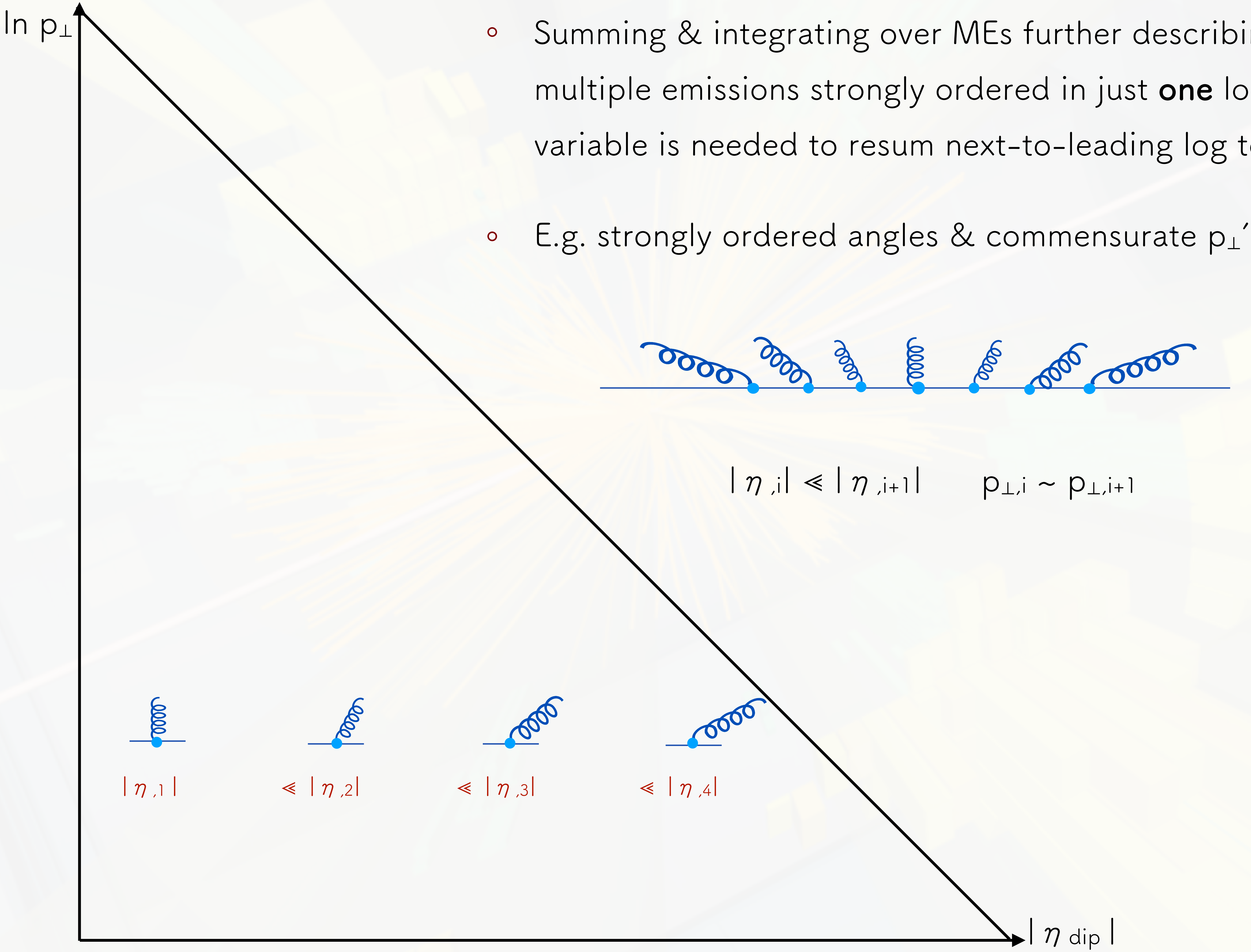


- Summing & integrating over MEs further describing multiple emissions strongly ordered in just **one** log variable is needed to resum next-to-leading log terms

- E.g. strongly ordered energies at commensurate angles

$E_{,i+1} \ll E_{,i} \qquad |\eta_{,i}| \sim |\eta_{,i+1}|$

leading logs : strongly ordered emissions in one logarithmic variable



- Summing & integrating over MEs further describing multiple emissions strongly ordered in just **one** log variable is needed to resum next-to-leading log terms

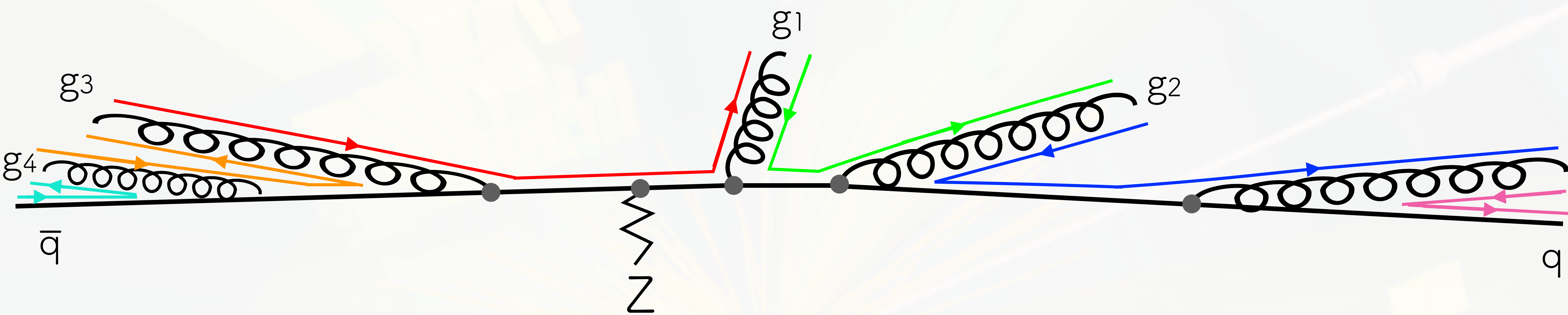
- E.g. strongly ordered angles & commensurate $p_\perp$'s

$|\eta_{,i}| \ll |\eta_{,i+1}| \qquad p_{\perp,i} \sim p_{\perp,i+1}$

NLL building blocks

NLL building blocks : ME for strong angular ordered [A.O.]

- NLL for global observables only requires correctly accounting for config^{n}s with strong A.O.



- Widest angle gluon is blind to smaller angle ones: thinks it was emitted from original $q\bar{q}$ dipole
- X-sec^{n} factorises into wide angle gluon from original dipole x that for the n-1 particle process
- Reasoning iterates on the resulting n-1, n-2, ... x-sec^{n}s on the same basis
- I.e. probability for n emissions widely separated in angle is just n lots of the one-emission prob:

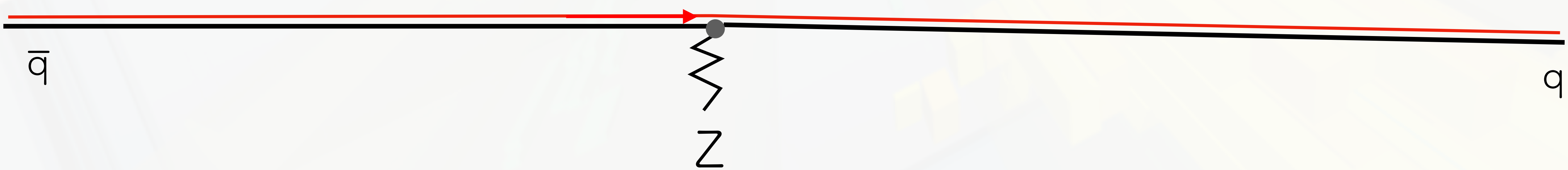
$$dP_n \simeq \frac{C_F^n}{n!} \prod_{i=1}^{n} \left(\frac{\alpha_s^{\text{CMW}}(p_{\perp,i}^2)}{\pi} \frac{dp_{\perp,i}}{p_{\perp,i}} dz_i P_{q\to qg}(z_i) \frac{d\phi_i}{2\pi} \right)$$

- Evaluating α_S at $p_\perp$ in the CMW scheme accounts for secondary emissions [inclusively]
- Holds for strong A.O. even if emissions have $p_{\perp,i} \sim p_{\perp,i+1}$ [PS recoil better respect this]

NLL building blocks : ME for strong energy ordering

- Non-global observables involve non-trivial partitions in phase space, e.g. jet-cone
- Resummation requires correct handling of emissions commensurate in angle, ordered in energy
- $E_{,i+1} \ll E_{,i}$ implies eikonal approximation
- In the large-Nc limit dipoles don't talk to each other [all charged differently]
- Ratio of n+1- to n-particle MEs for a given colour config^{n} is sum of MEs for each dipole to emit

$$dP_{q\bar{q}} \propto \frac{q.\bar{q}}{q.g_1 \; \bar{q}.g_1}$$



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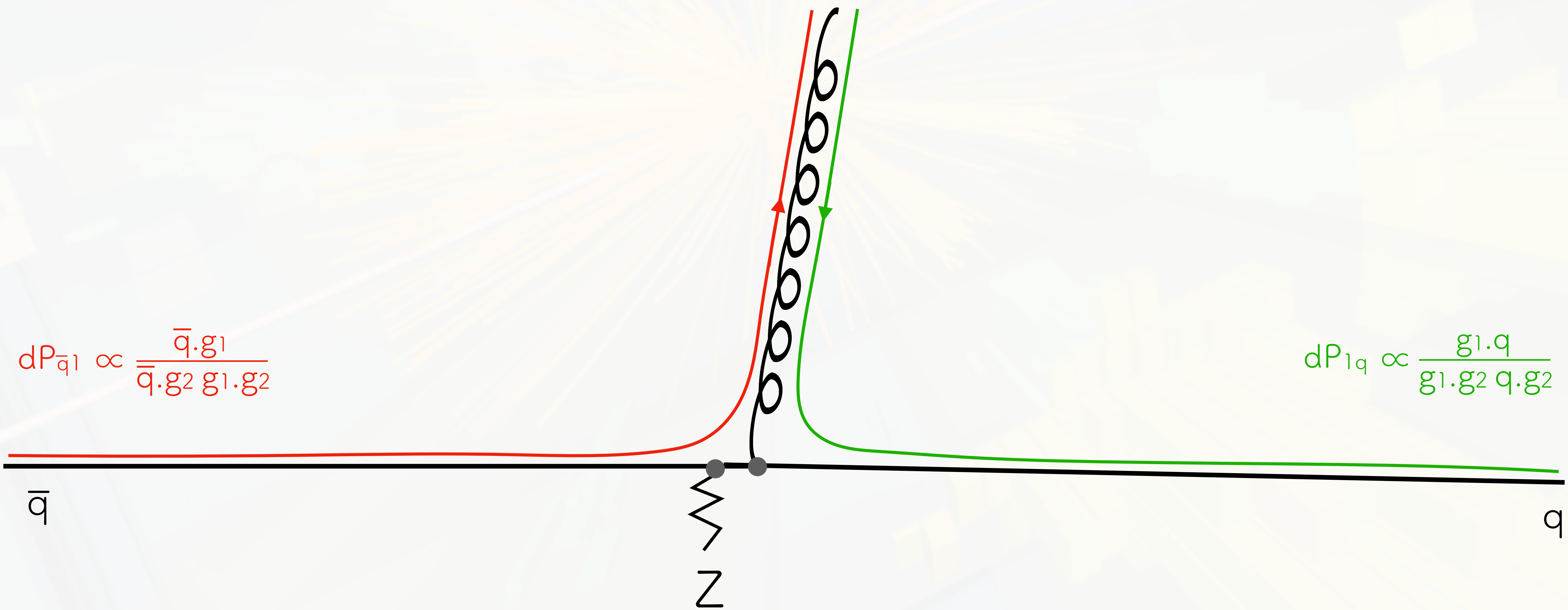

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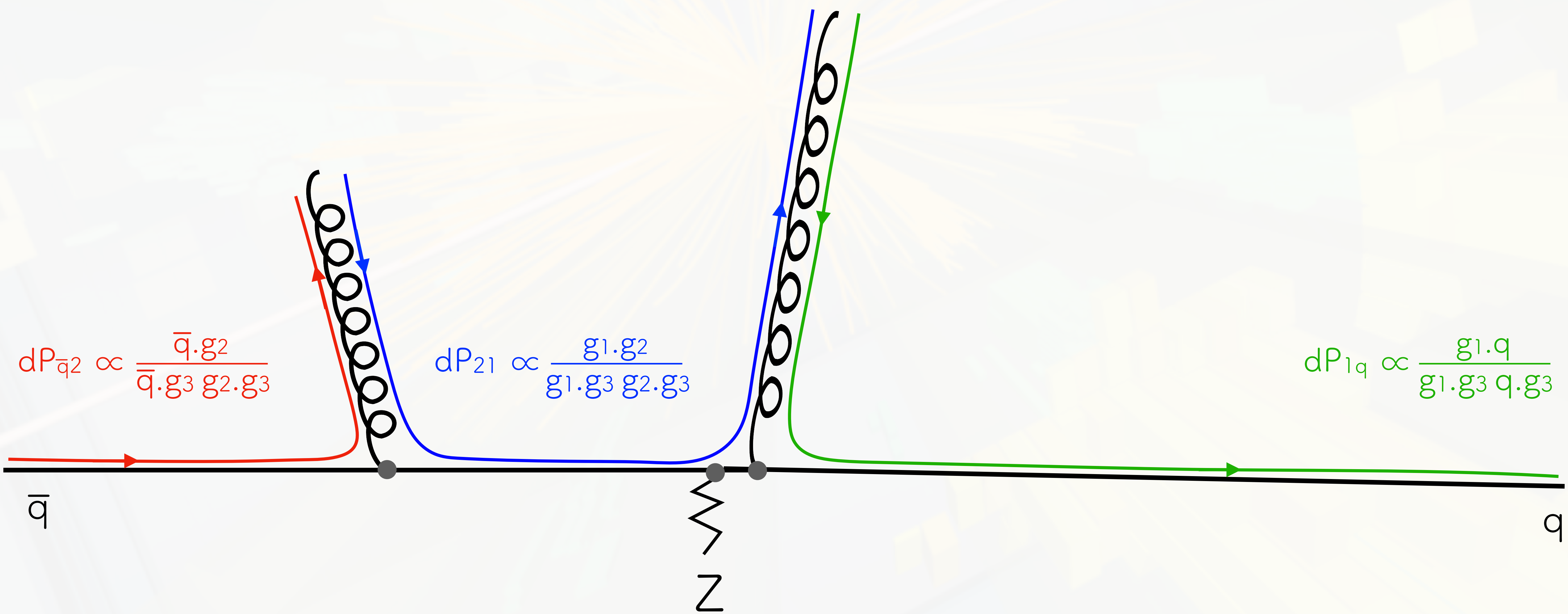

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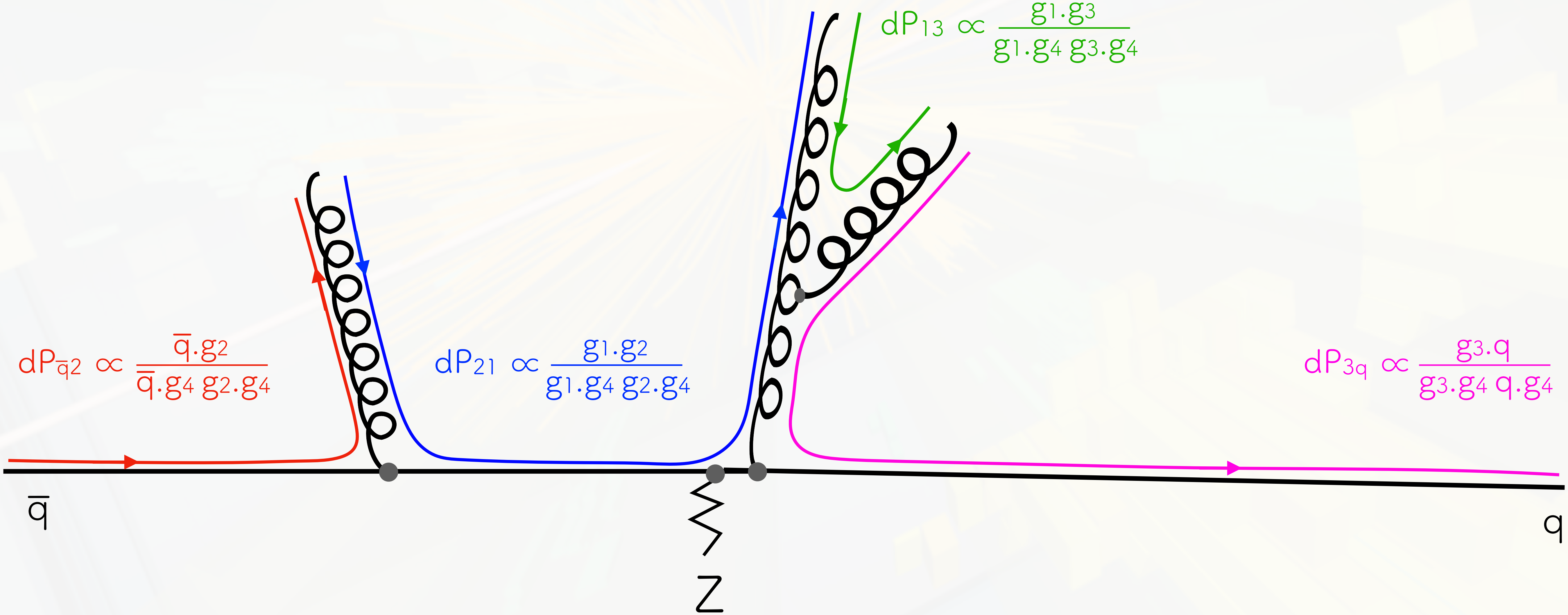

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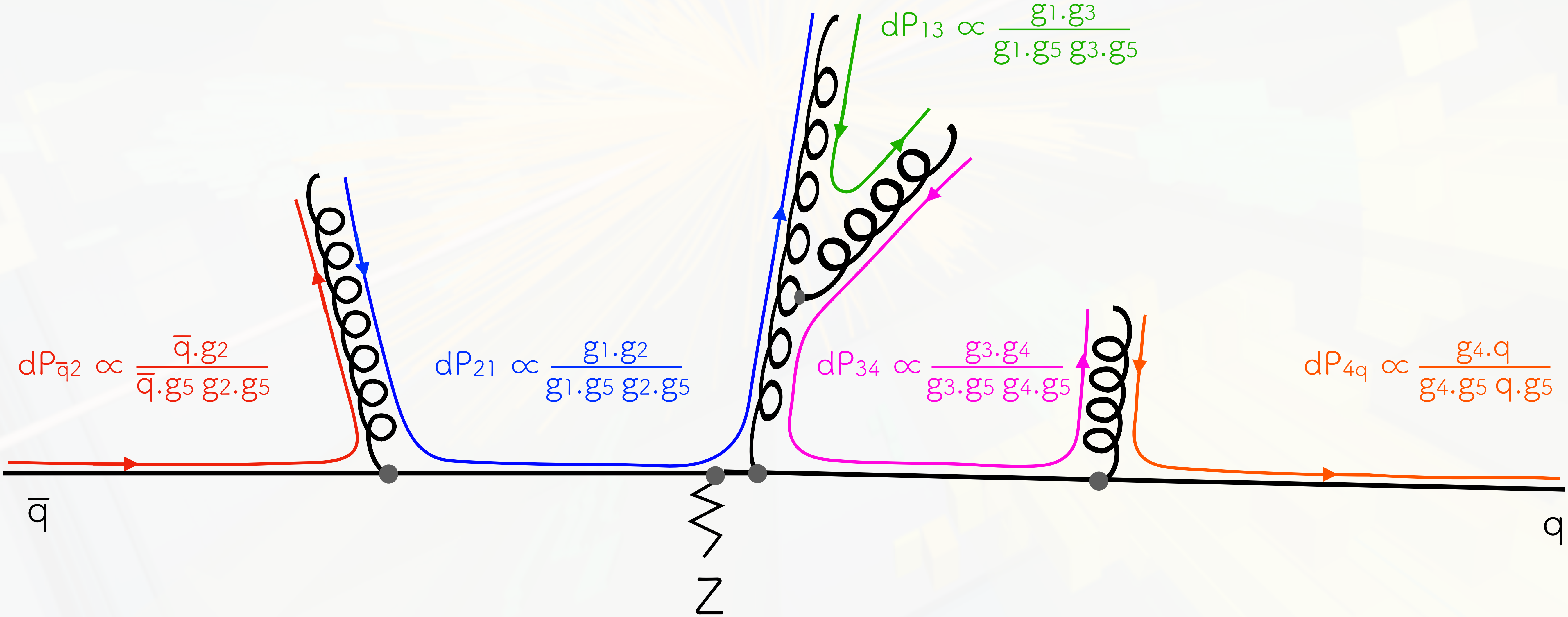

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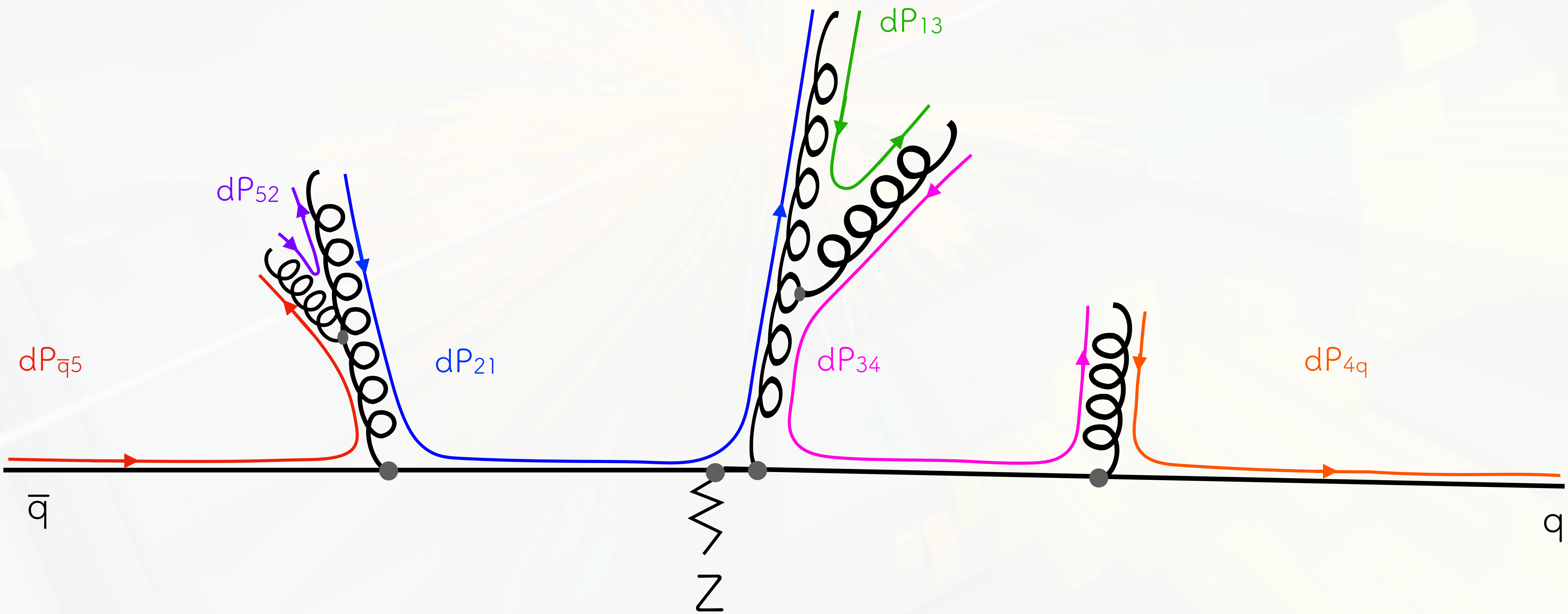

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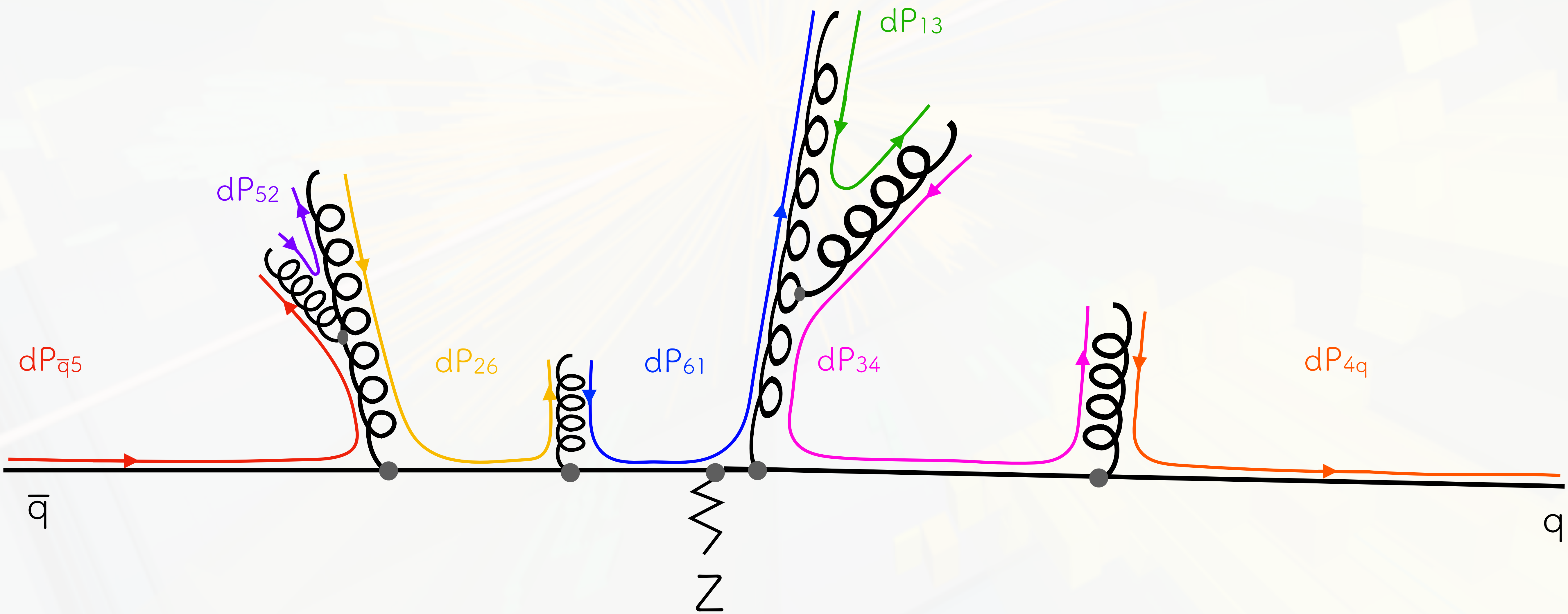

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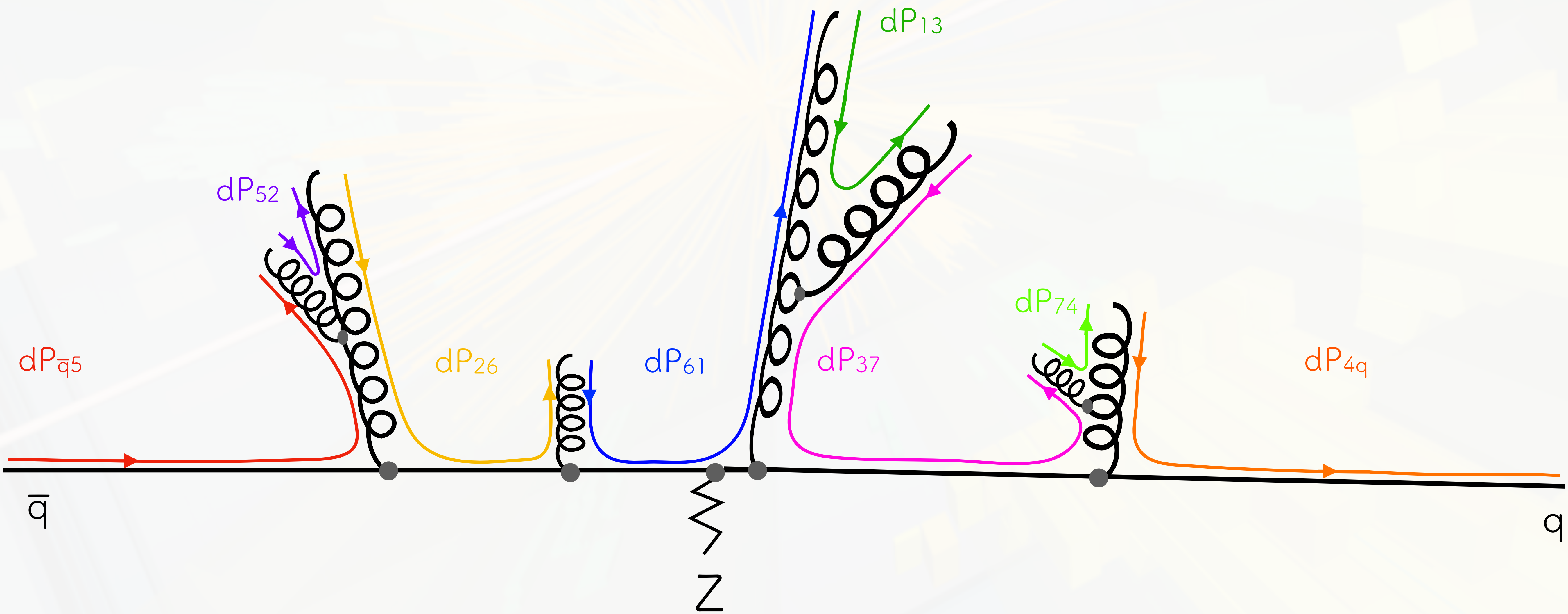

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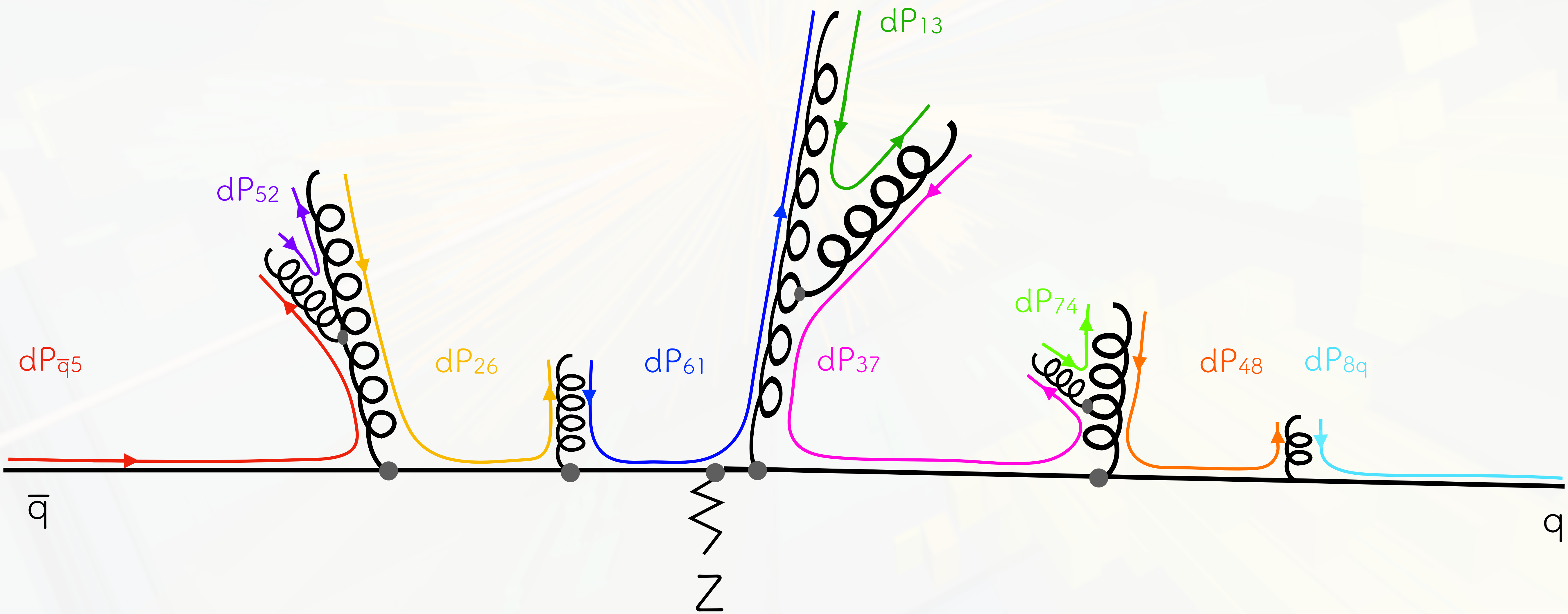

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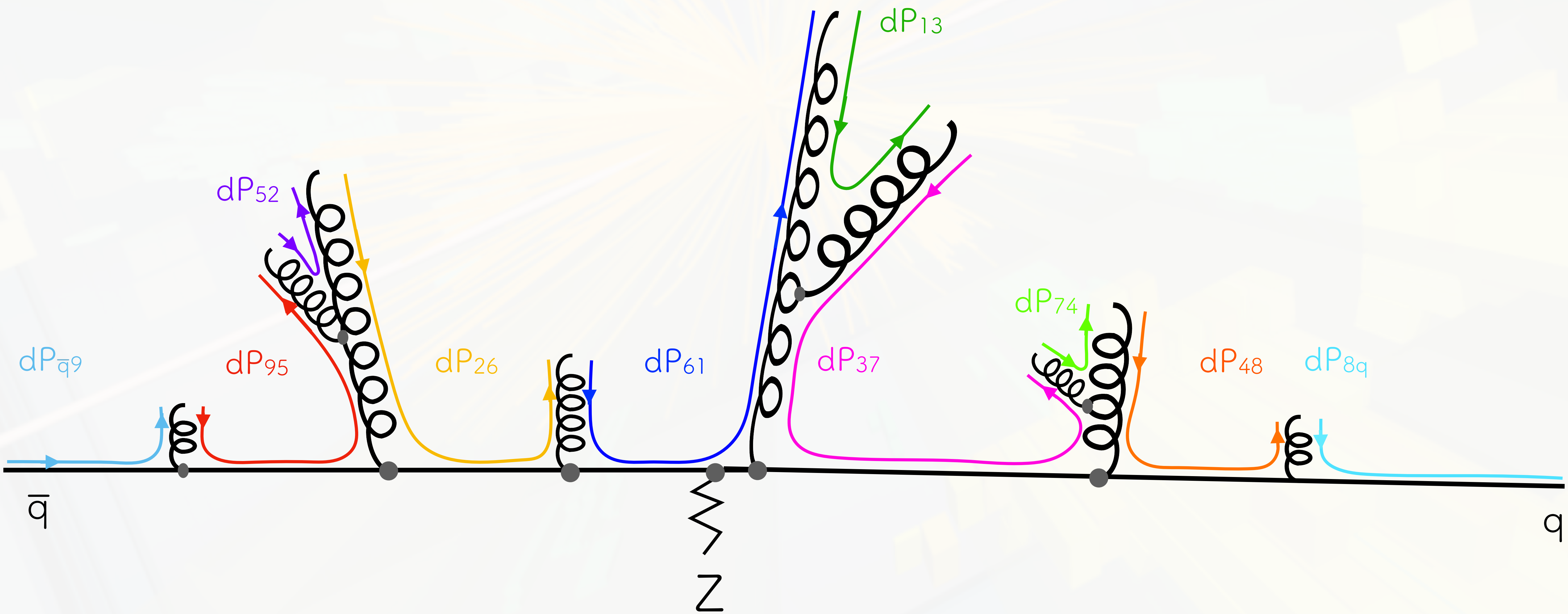

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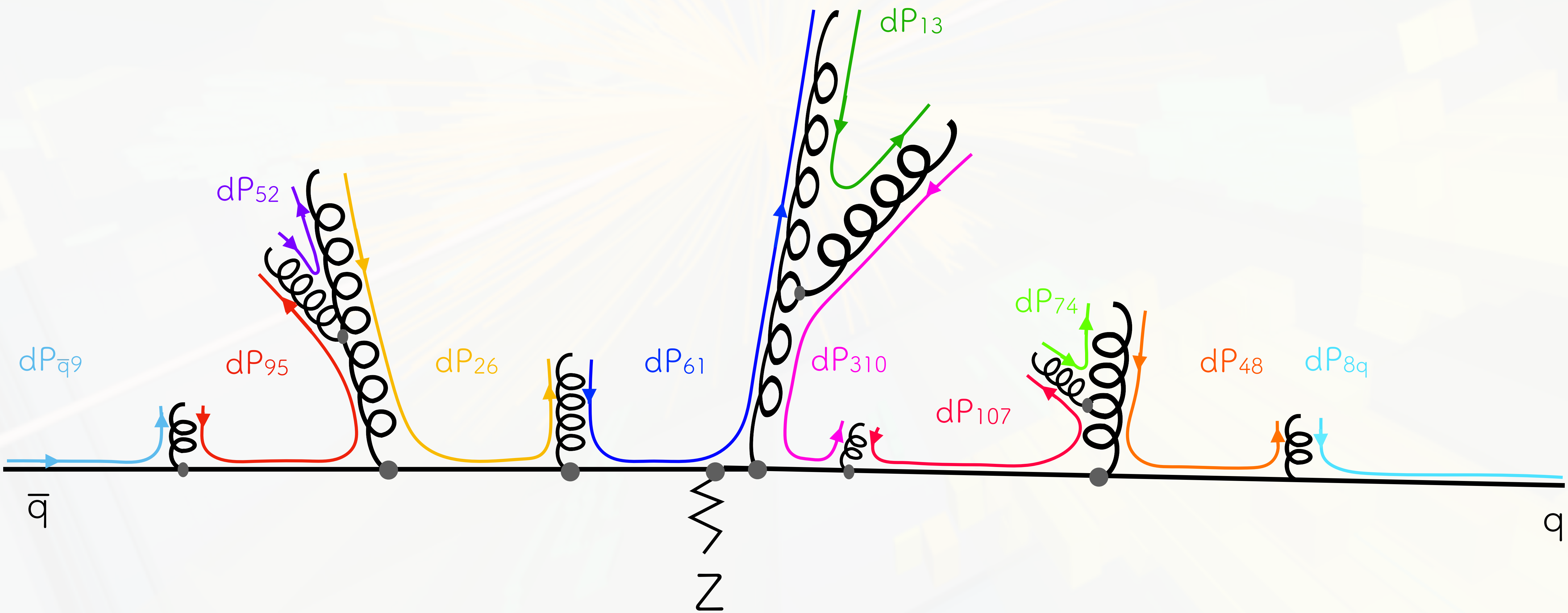
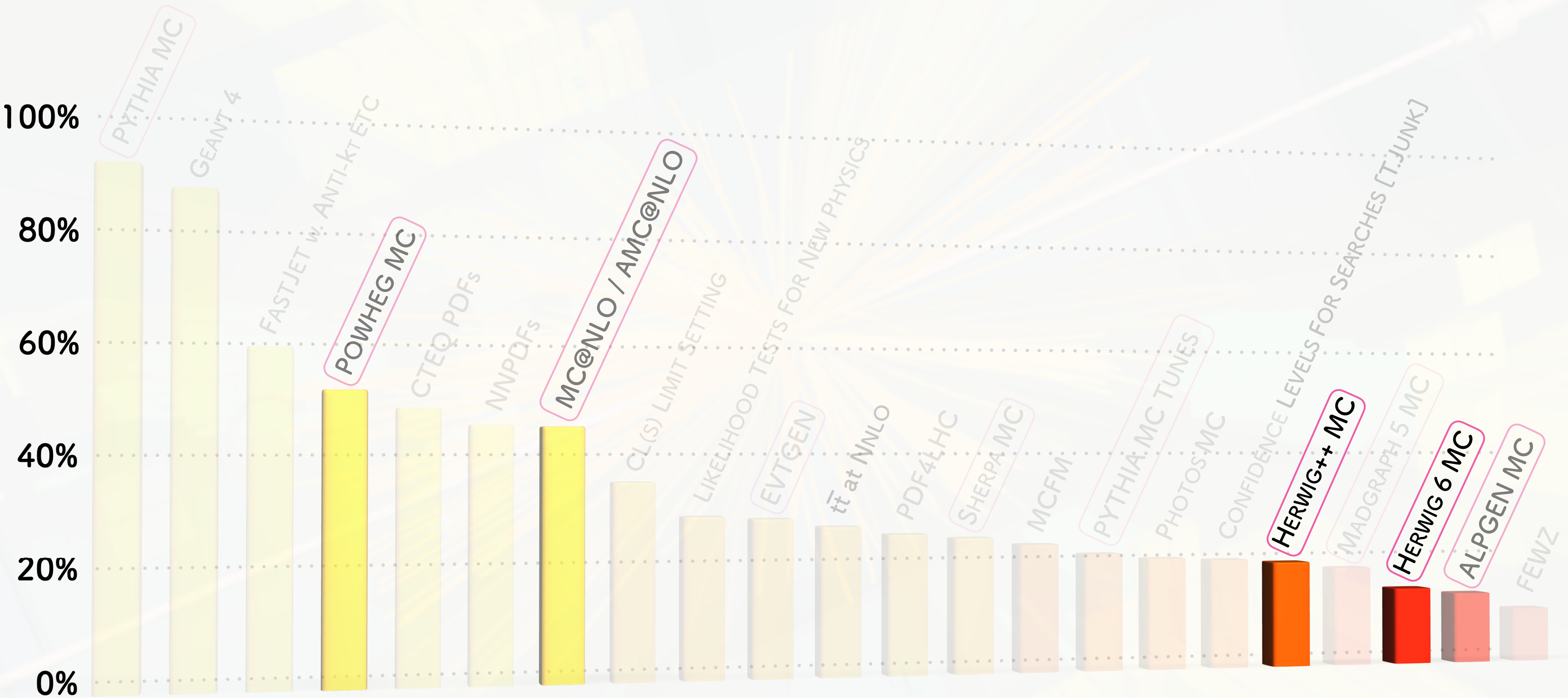


existing parton showers

Showers built out of the strong angular ordering picture [A.O.]

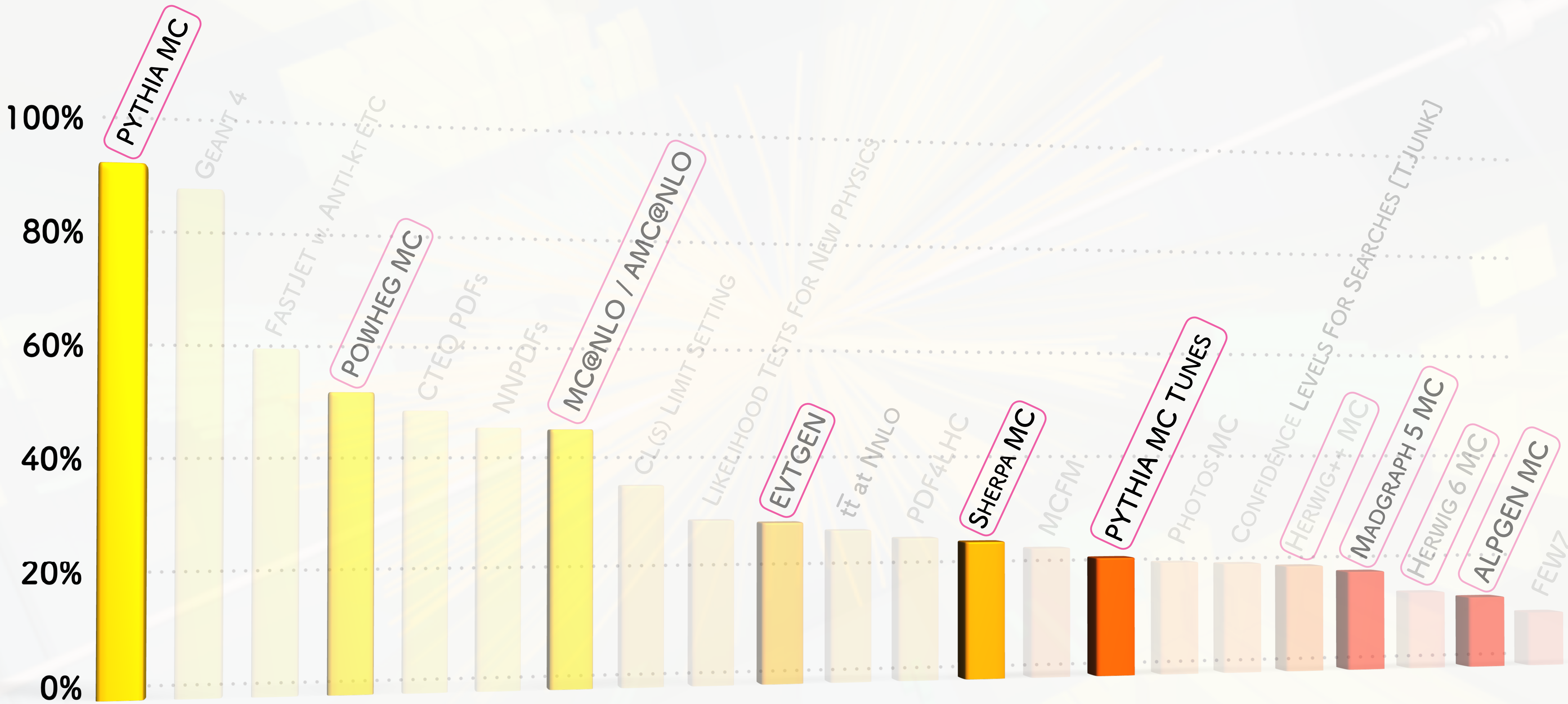
- Only the HERWIG/HERWIG++ PS MCs take strong A.O. as their core construction principle [feeding into NLO+PS tools producing hard scattering configurations in need of showering]



- It was clear since the first days of NGLs angular ordering, on its own, won't resum them
- Banfi, Corcella, Dasgupta '06 studied this for HERWIG, showing discrepancies with LL NGL calc^{n}s

Showers built out of the dipole picture

- PYTHIA, DIRE, SHERPA have the dipole multiplication model as their core construction principle†
- They additionally match the soft dipole f^{n}s to AP splitting f^{n}s to describe hard-collinear rad^{n} ...



- Foundations to start building the next generation of precision PS MC from?
- No. Turns out they fundamentally don't reproduce the A.O. limit — arXiv:1805.09327

† ARIADNE [long long ago] was the first PSMC based on the dipole picture [Gustafson, Lönnblad & others]

current dipole PS MCs & QCD coherence

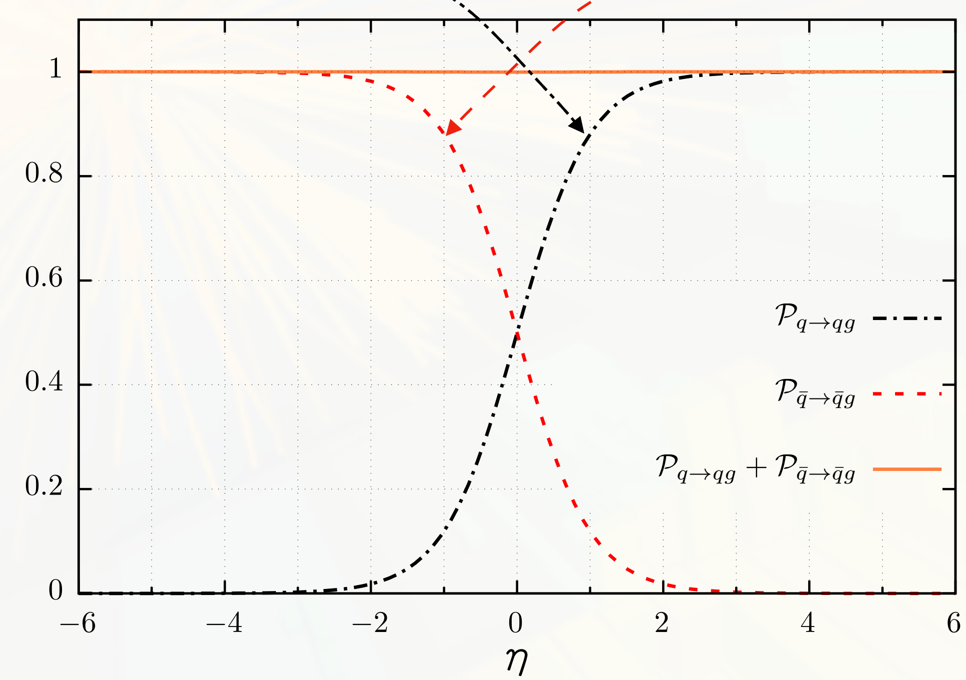
dipole PSMCs partition rad^{n} pattern w.r.t the colour & anti-colour ends

- On-the-market dipole PSMCs split eikonal f^{n}s up symmetrically in the **dipole C.O.M**

$$d\mathcal{P}_{\tilde{i}\tilde{j}\to ikj} = d\mathcal{P}_{\tilde{i}[\tilde{j}]\to ik[j]} + d\mathcal{P}_{\tilde{j}[\tilde{i}]\to jk[i]}$$

$$= \frac{2\alpha_{\rm S}C}{\pi}\frac{dp_\perp}{p_\perp}d\eta \boxed{\frac{e^{2\eta}}{1+e^{2\eta}}} + \frac{2\alpha_{\rm S}C}{\pi}\frac{dp_\perp}{p_\perp}d\eta \boxed{\frac{e^{-2\eta}}{1+e^{-2\eta}}}$$

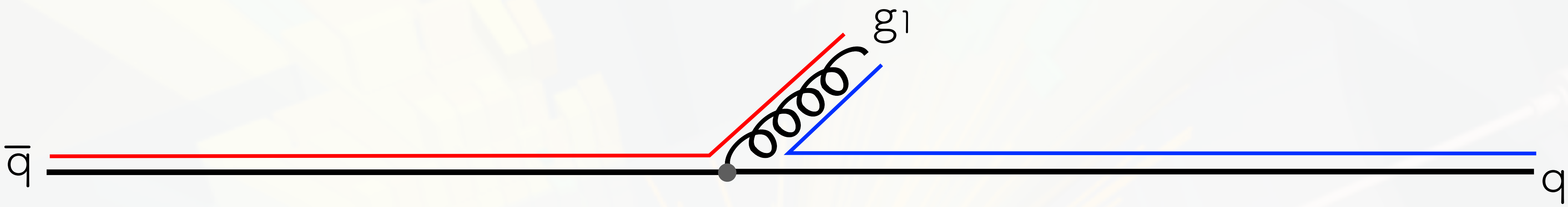
$$= \frac{2\alpha_{\rm S}C}{\pi}\frac{dp_\perp}{p_\perp}d\eta$$



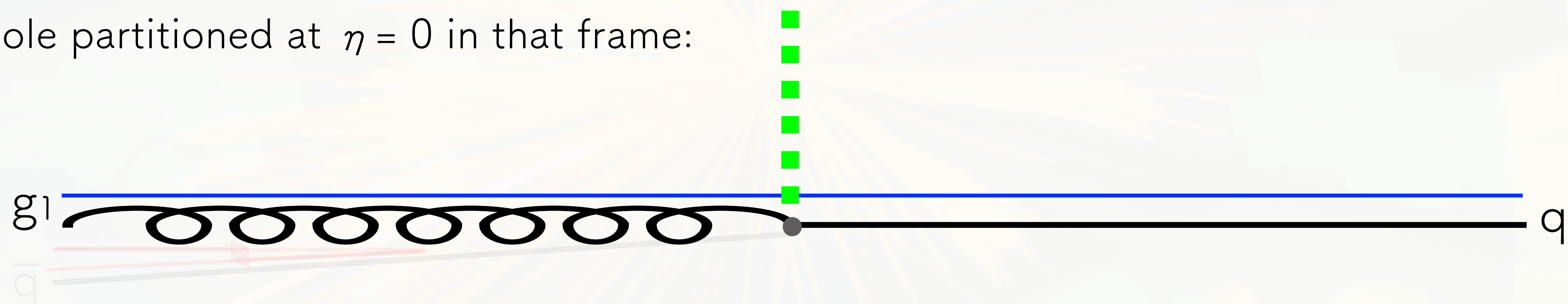
- $\tilde{\imath}$ end accounts for full dipole branching probability in limit i $\parallel$ k [$\eta \to +\infty$]
- $\tilde{\jmath}$ end accounts for full dipole branching probability in limit j $\parallel$ k [$\eta \to -\infty$]

dipole partitions back in the event COM are not symmetric

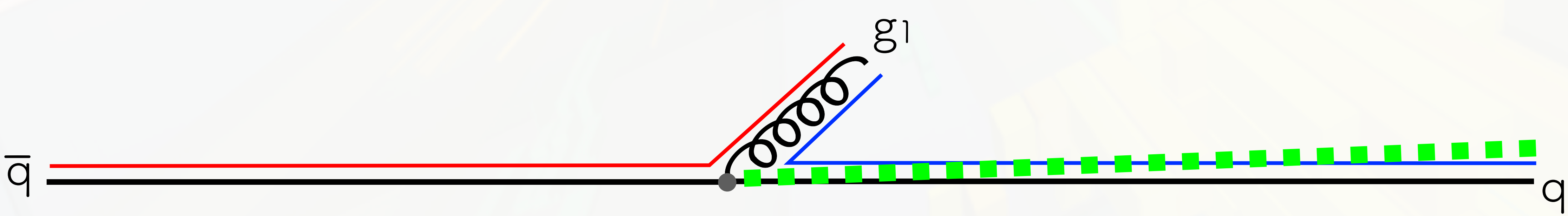
- Consider we emitted **soft gluon g_1** from **hard** $q\bar{q}$, so we end up with a $\bar{q}g_1$ and a g_1q dipole:



- To get us from the event COM to the g_1q dipole COM [blue line] requires a **BIG BOOST** ⟶
- Dipole partitioned at $\eta = 0$ in that frame:



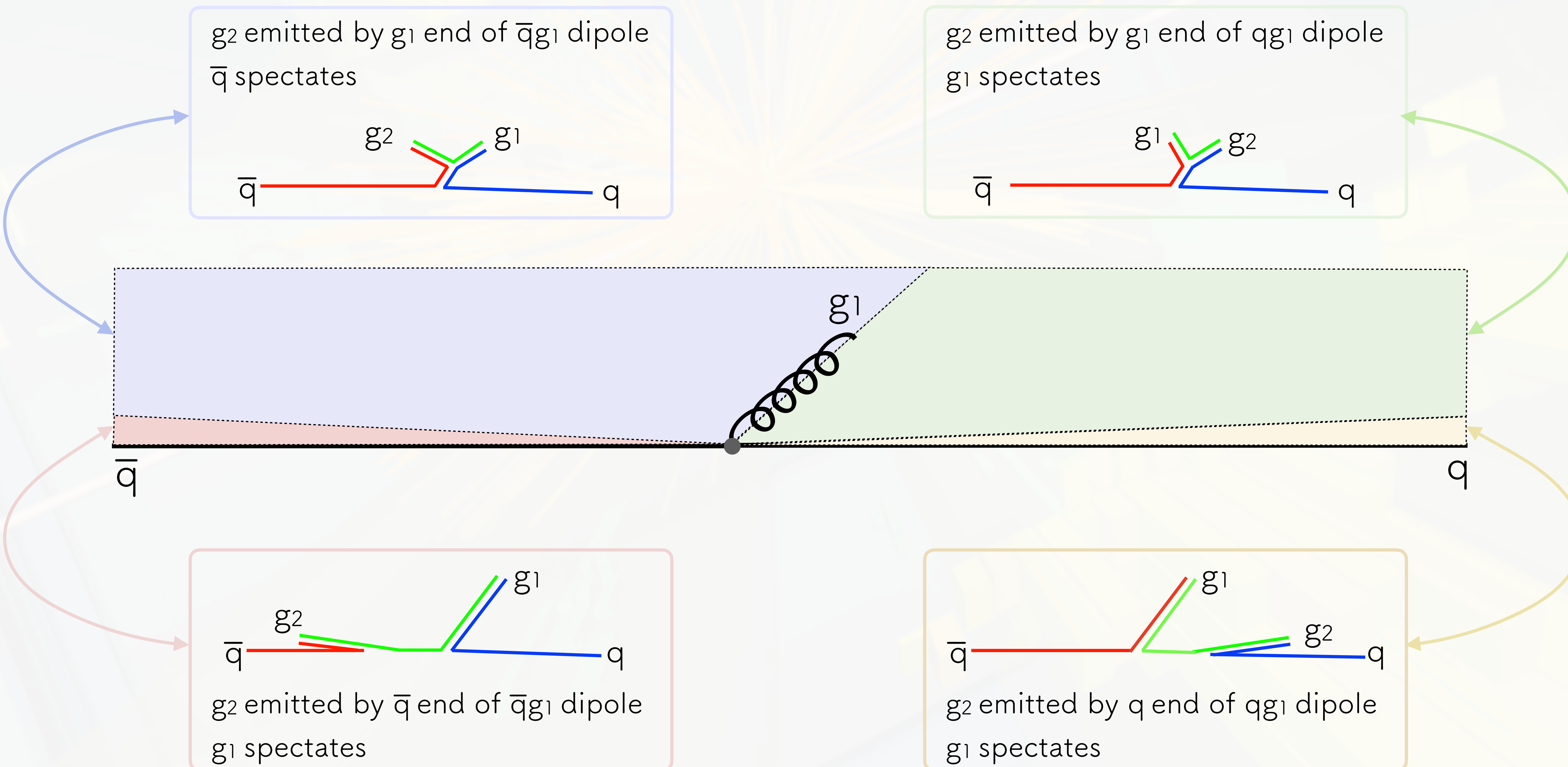
- To get us back to the event COM from the g_1q dipole COM undo the same **BIG BOOST** ⟵



- In event COM partition comes out very close to q ; instead of equidistant in angle between g_1 & q

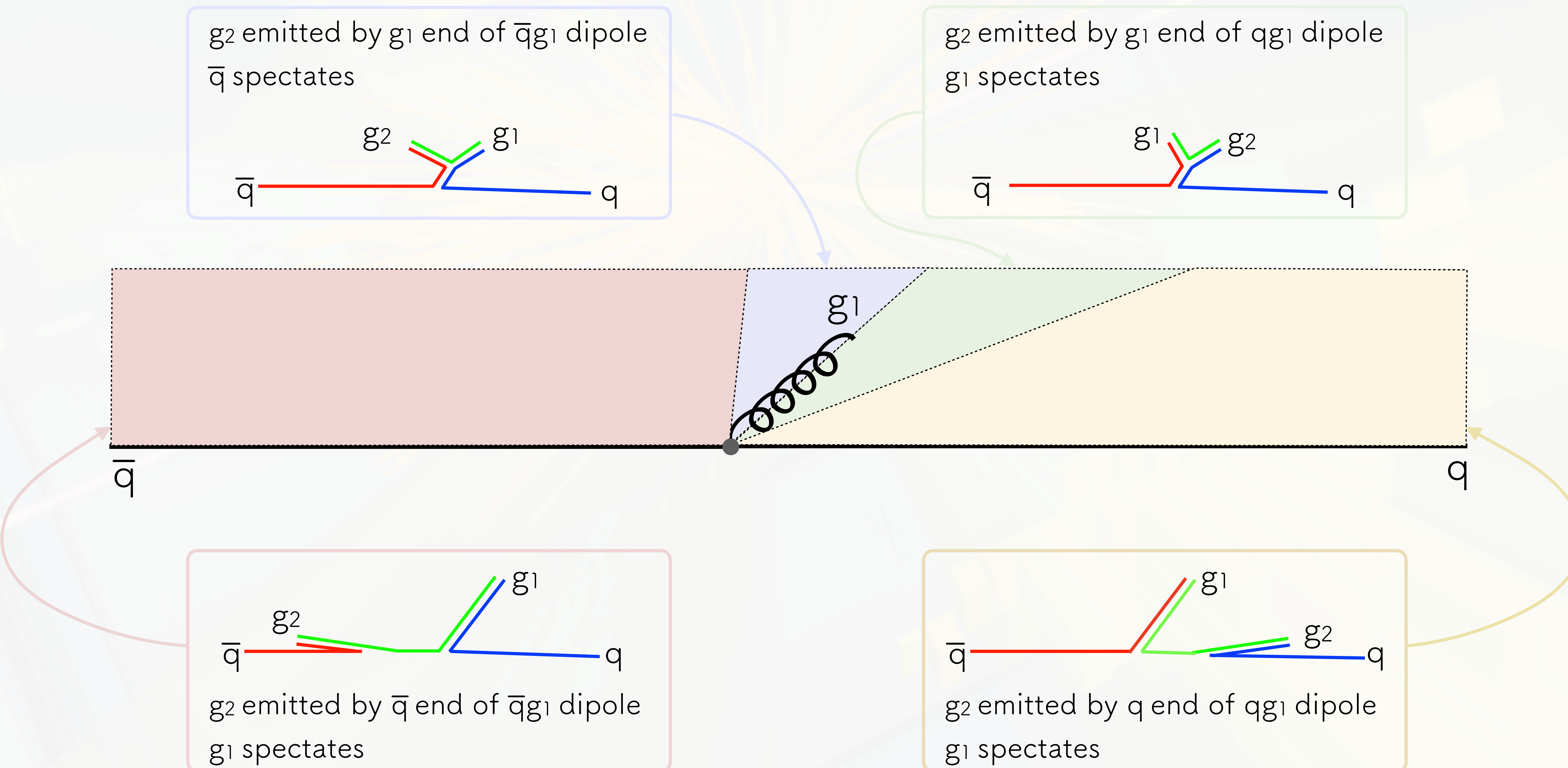
dipole partitions back in the event COM are not symmetric

- $\bar{q}g_1$ dipole partition is similarly located at $\eta = 0$ in $\bar{q}g_1$ COM
- But for soft g_1 the $\bar{q}g_1$ COM also involves **BIG BOOST** $\longrightarrow$ to return that back to event COM
- Our dipole PSs thus encode the following partitioning & associated emitter-spectator labelling



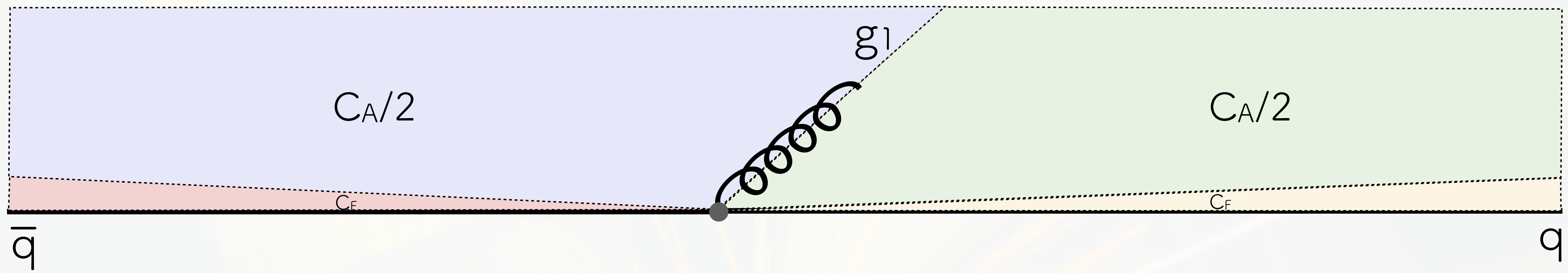
dipole partitions that angular ordering would prescribe

- Colour coherence [angular ordering] instead dictates we should partition things more like in the following picture, using the same colour coding for g_2's emitter-spectator assignment as before

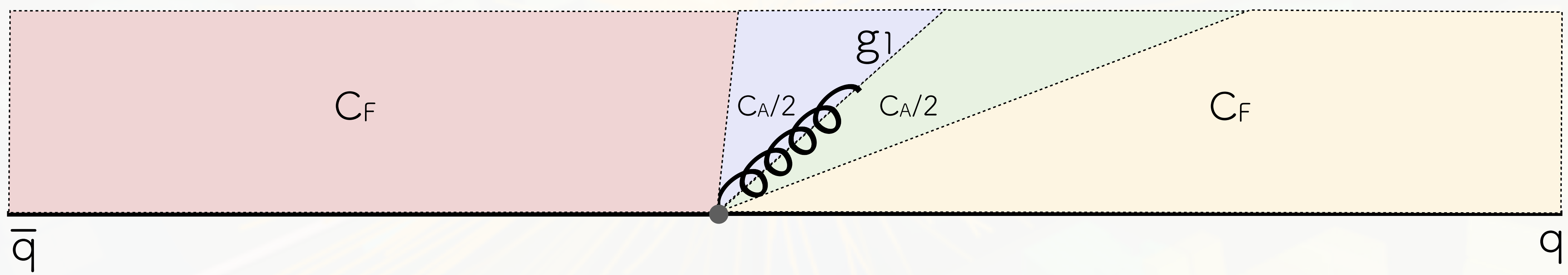


dipole partitions and colour factor issues

- The dipole shower phase space partitioning of g_2's radiation pattern is:



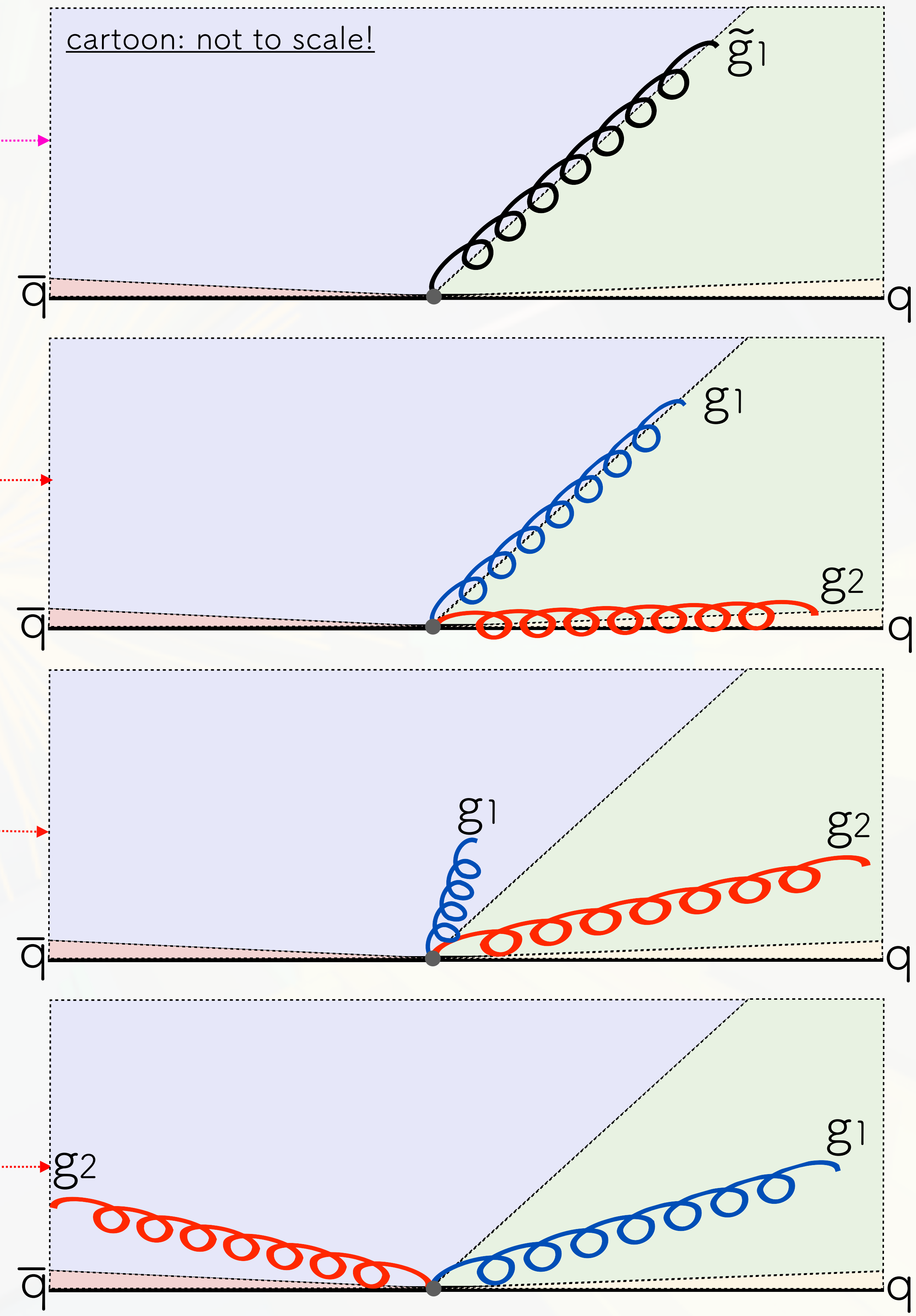
- Angular ordering implies a partitioning more like the following:



- In attributing emission of g_2 to g_1 over much greater angular regions than advocated by colour coherence — including regions when g_2 is essentially collinear to the q or $\bar{q}$ — our dipole showers generate g_2 with a $C_A/2$ colour factor in regions where the correct colour factor is C_F
- The effective soft+collinear ME for g_2 has wrong colour factor including at small angles
- In general causes spurious subleading N_C terms to appear at LL level

dipole partitions and recoil attribution issues

- Emitter-spectator assignment also determines how recoil is attributed
- Emitter balances $p_\perp$ of emission w.r.t emitter's original dir^n, spectator's energy shrinks to make up the rest
- Consider $\tilde{g}_1$ on the top-right & how it recoils to become g_1 depending on the partition g_2 comes out in [depending on emitter assignment]
- Emissions widely separated in angle aren't supposed to talk to each other!
- $\tilde{g}_1$ kicked by g_2 too often
- This issue generalises to all orders leading to spurious NLL contributions

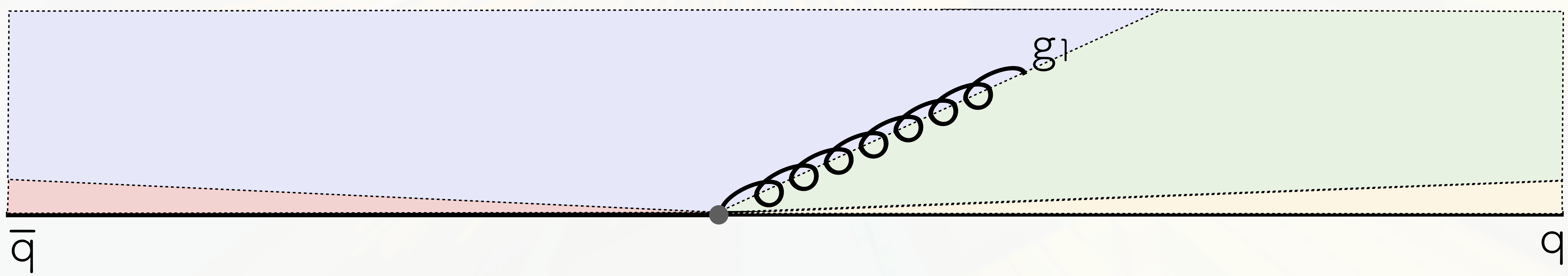


Mrinal Dasgupta, Frédéric Dreyer, KH, Pier Monni, Gavin Salam

PSMCs at NLL

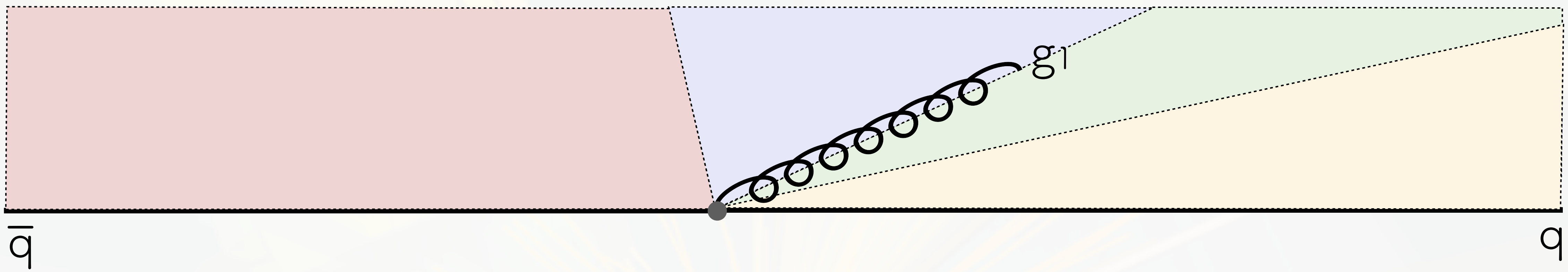
NLL PSMC with a dipole-local recoil scheme : PanLocal

- Existing dipole shower algorithms are correct for commensurate angles and ordered energies
- We seek to make them work also for emissions with commensurate $p_{\perp}$'s and strong A.O.
- In the first case we limit ourselves to the same dipole-local recoil employed in existing PSMC
- Recall existing dipole PSMCs partition dipoles symmetrically in angle in the dipole's C.O.M

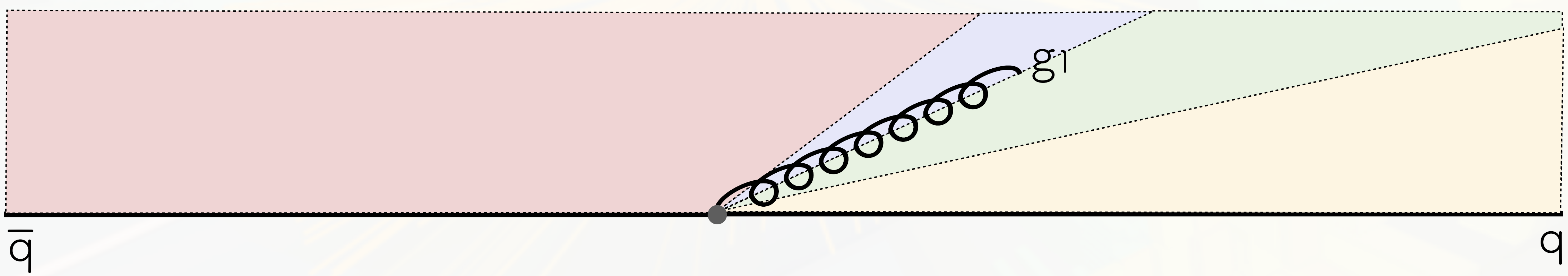


NLL PSMC with a dipole-local recoil scheme : PanLocal

- **Novel element 1** : partition each dipole symmetrically in angle in the **event C.O.M** instead:



- Better but still not as A.O. prescribes in the $\bar{q}g_1$ dipole region



- Rad^n can be emitted at wide angles w.r.t earlier emissions [here g_1] with recoil, colour factors, etc all still attributed to the wrong emitter: spurious LL large-Nc terms & NLL full Nc ones

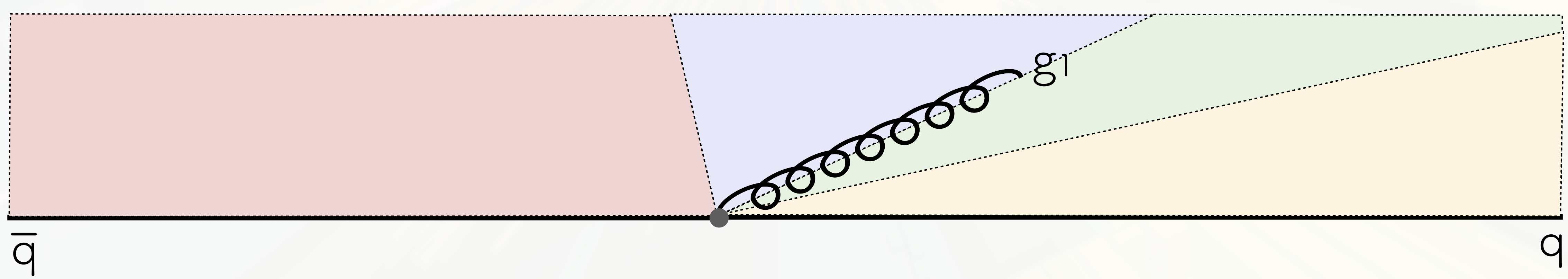
NLL PSMC with a dipole-local recoil scheme : PanLocal

- **Novel element 2** : choose an evolution variable that effectively imposes some angular ordering

$$v \sim k_{t,\tilde{i}k}\, e^{-\beta|\eta_{\tilde{i}k}|} \sim k_{t,\tilde{i}k}\, \theta_{\tilde{i}k}^{\beta} \qquad [\, 0 < \beta < 1\,]$$

- Ordering emissions in this variable implies those with commensurate k_t's are ordered from larger to smaller angles

- Any later emissions going in the wide angle part of the blue region have at least comparable emission angles to g_1 : ordering in v then implies $k_{t,1} \gg k_{t,2}$, i.e. there is no recoil to mishandle



- Together, the dipole partitioning and ordering variable combine such that at any significant k_t recoil in the event is always taken by the extremities of the [hard] $qg\{\cdots\}g\bar{q}$ dipole chain

NLL tests of full-fledged PS MC implementations

Testing NLL accuracy : global and non-global observables

- We considered cumulative distributions for a wide variety of observables, e.g. jet rates and event shapes, in the limit $\alpha_S L \sim 1$ and $|L| \gg 1$, with 2-loop running α_S in the CMW scheme

- Results for all such tested observables are known from analytic resummation to have this form

$$\Sigma(\alpha_s, \alpha_s L) = \exp\Big[\underbrace{\alpha_s^{-1} g_1(\alpha_s L)}_{\text{LL} \sim \alpha_S^n L^{n+1} \sim O(\alpha_S^{-1})} + \underbrace{g_2(\alpha_s L)}_{\text{NLL} \sim \alpha_S^n L^n \sim O(1)} + \underbrace{\mathcal{O}(\alpha_s^n L^{n-1})}_{\text{NNLL} \sim \alpha_S^n L^{n-1} \sim O(\alpha_S)}\Big]$$

frac^n of events where observable value < e^L

zero for η slice

- To compare PSMC to analytic NLL we compute ratios $\Sigma_{PS} / \Sigma_{NLL}$ and extrapolate to $\alpha_S \to 0^\dagger$

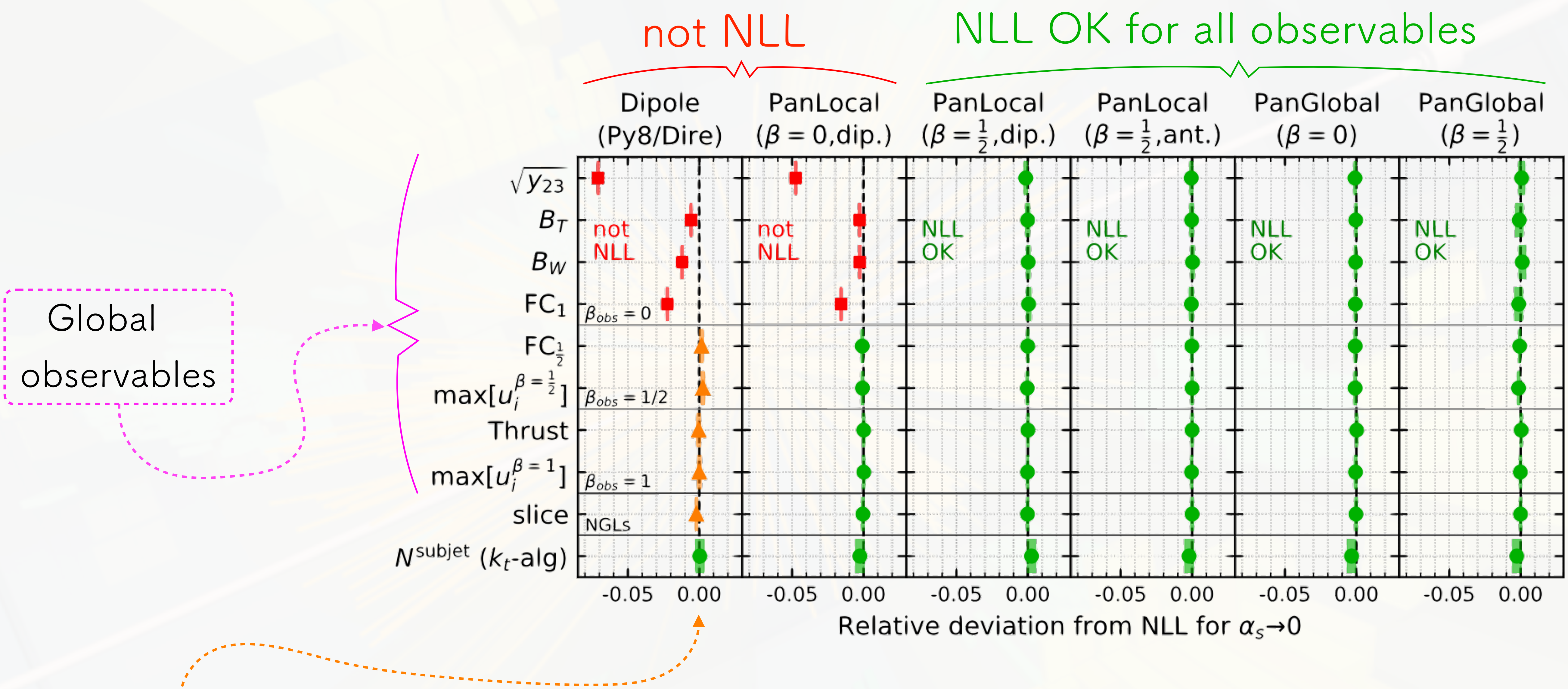
  - If PS **not NLL** , Σ_{PS} fails for $O(\alpha_S^n L^n)$ i.e. $O(1)$ we will find $\lim_{\alpha_S \to 0} \dfrac{\Sigma_{PS}}{\Sigma_{NLL}} \neq 1$

  - If PS **NLL OK** , Σ_{PS} fails for $O(\alpha_S^n L^{n-1})$ i.e. $O(\alpha_S)$ we will find $\lim_{\alpha_S \to 0} \dfrac{\Sigma_{PS}}{\Sigma_{NLL}} = 1$

† This technical challenge well outweighs the theoretical one of formulating the new PSMC models

Testing NLL accuracy : comparison of PSMCs to dedicated NLL calcns

- Global observables : testing the angular ordered regime

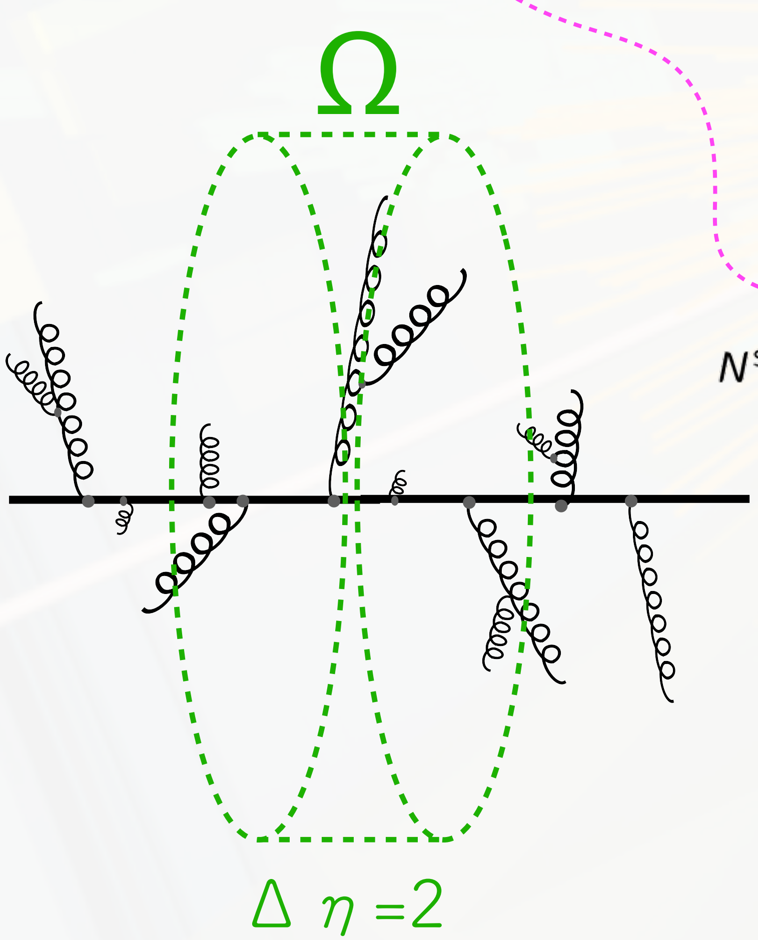


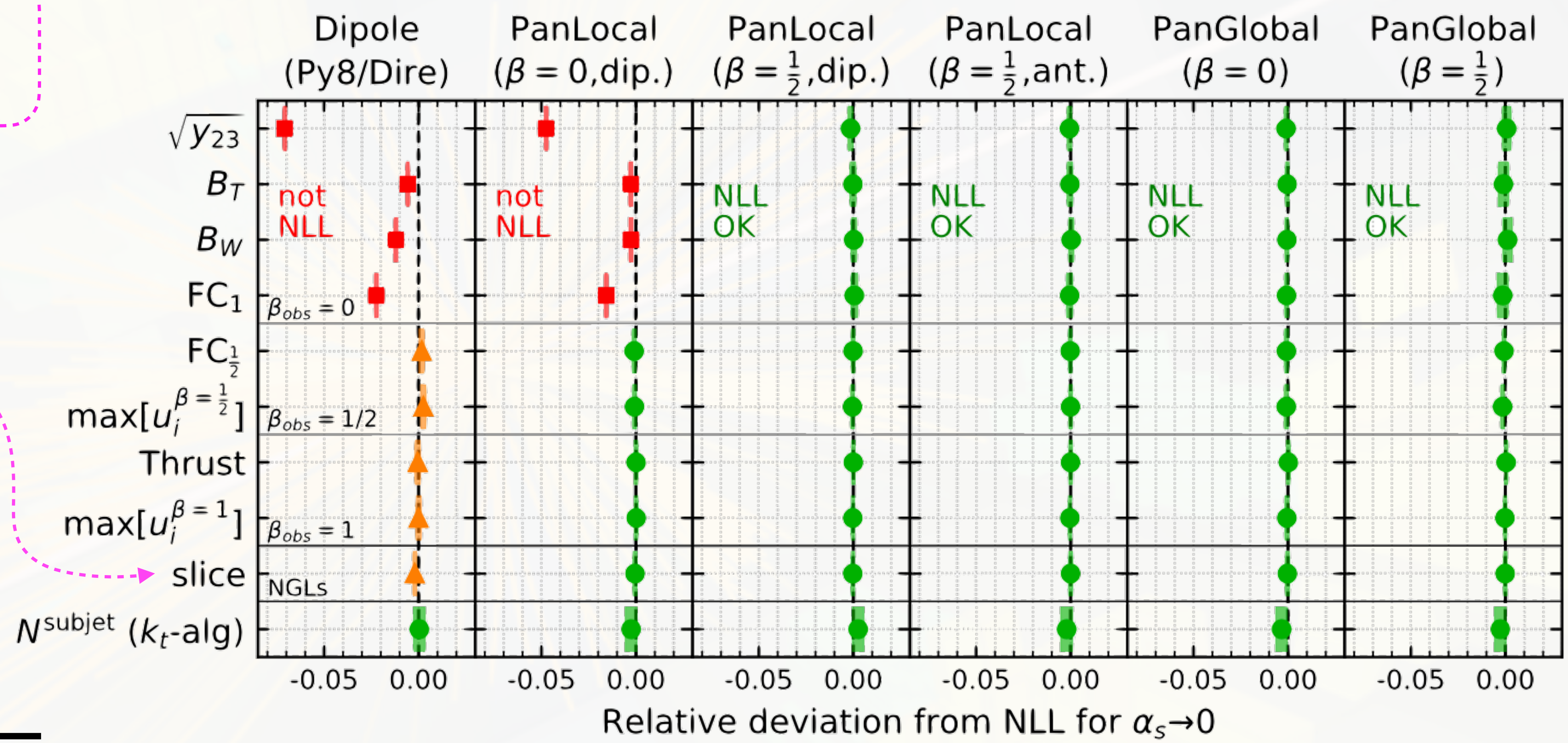
- Orange triangles signal that fixed order analysis reveals deviations from NLL results
- As expected, PanLocal with transverse momentum ordering [$\beta = 0$] fails like PYTHIA8/DIRE

Testing NLL accuracy : comparison of PSMCs to dedicated NLL calc^{ns}

- Non-global observables : testing the energy ordered commensurate angle regime

non-global: scalar p_t sum in rapidity slice



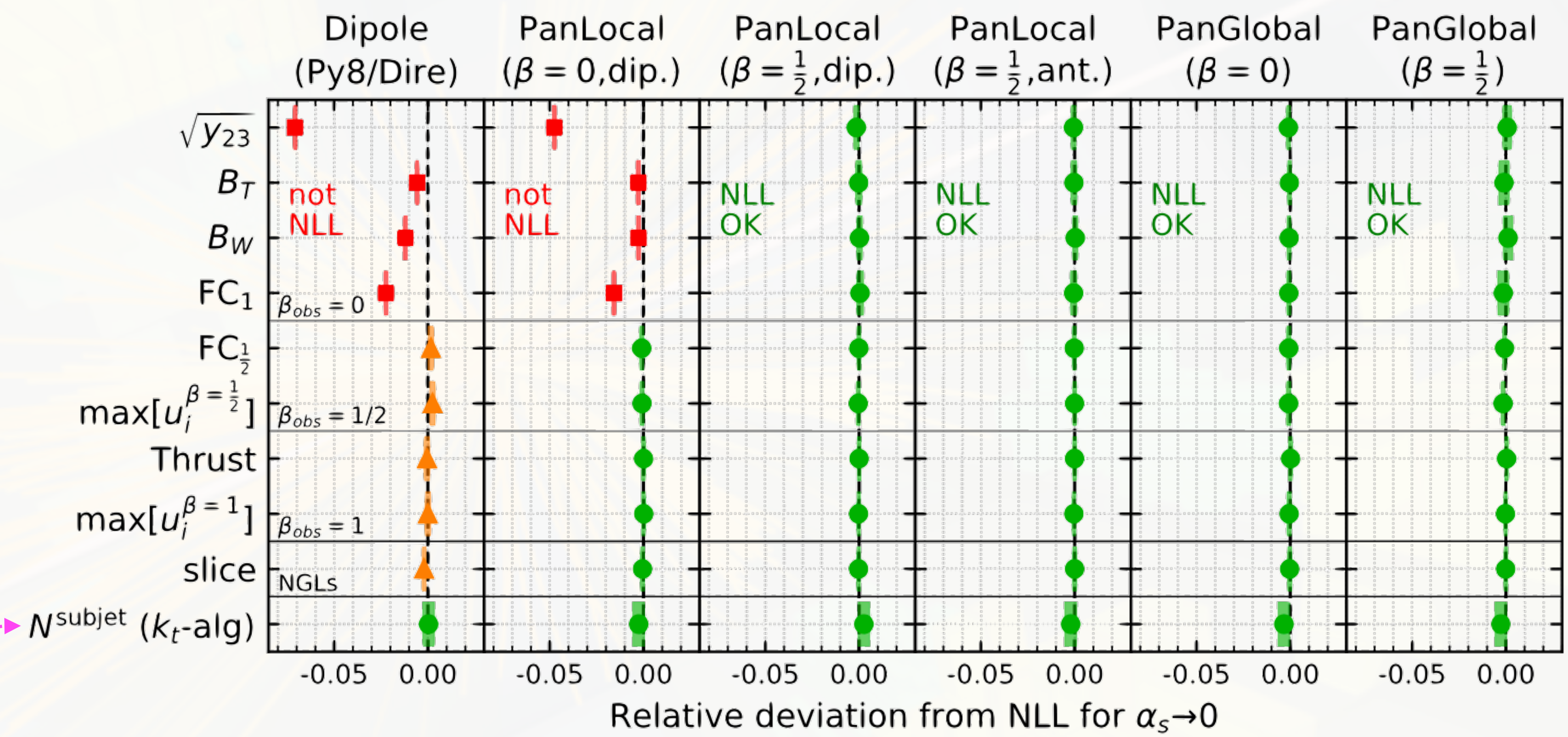


$$E_T = \sum_{i \in \Omega} E_{t,i}$$

Testing NLL accuracy : comparison of PSMCs to dedicated NLL calc^{ns}

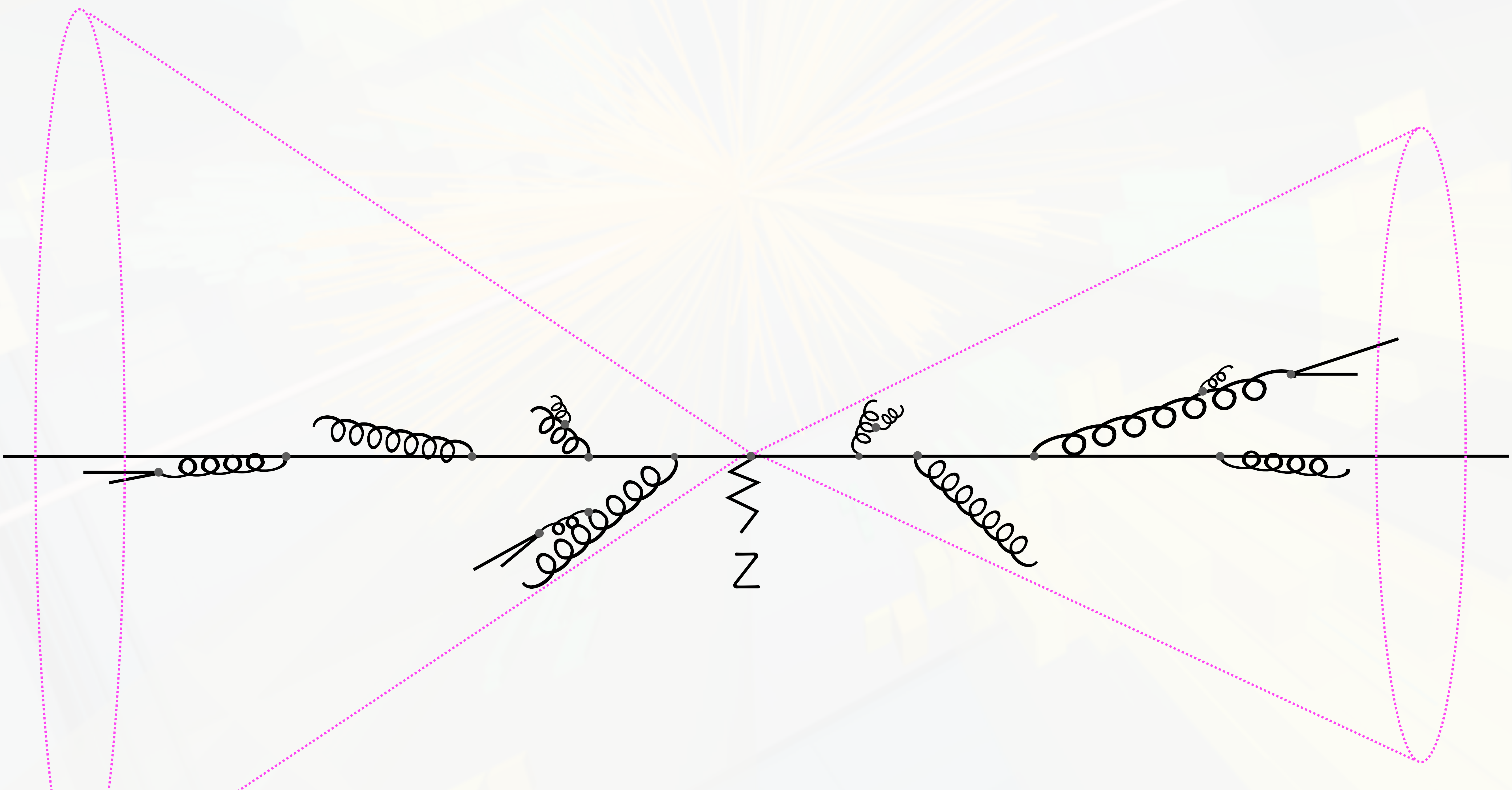
- k_t-jet multiplicity [as f^n of ln y_{cut}] : sensitive to full nested soft-collinear branching structure

k_t-jet multiplicity
[as f^n of ln y_{cut}]



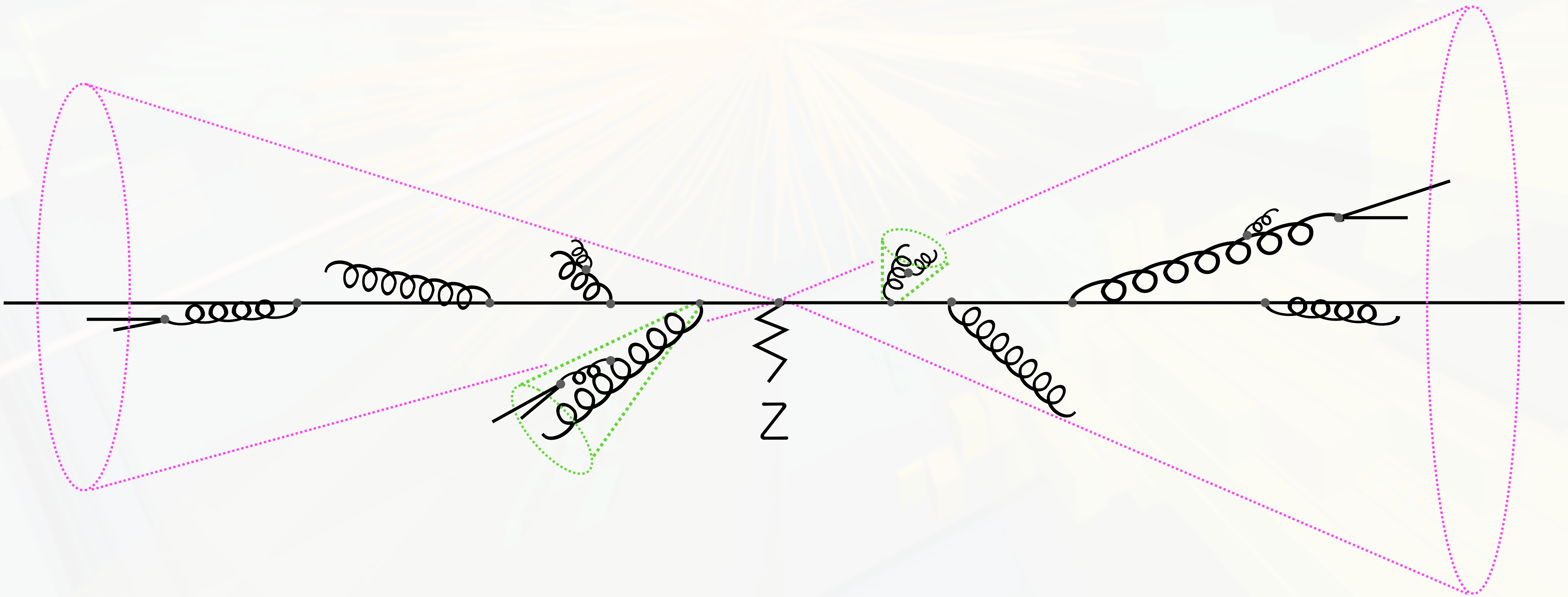
Testing NLL accuracy : multiple emission matrix elements

- Imagine the event is a real event as from mother nature
- Imagine it's clustered into two-jets by the Cambridge algorithm



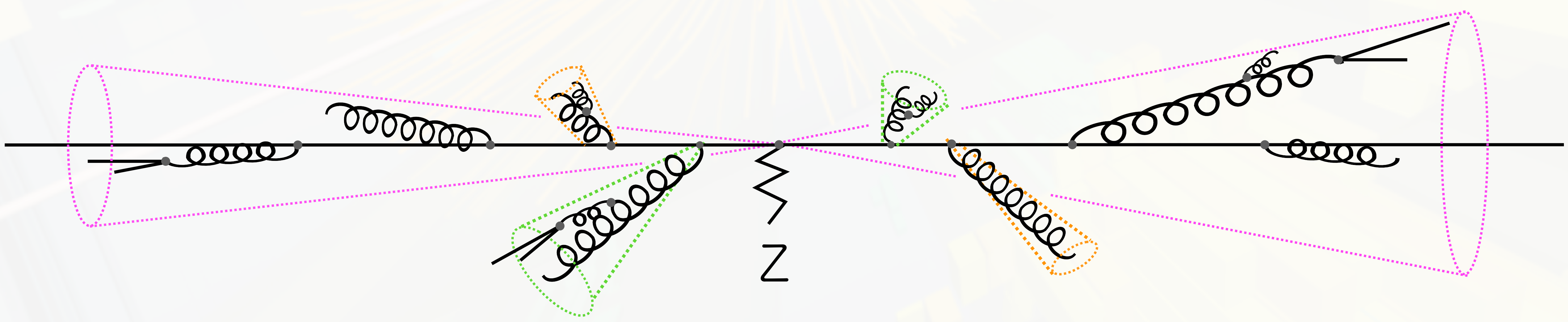
Testing NLL accuracy : multiple emission matrix elements

- Imagine the event is a real event as from mother nature
- Imagine it's clustered into two-jets by the Cambridge algorithm
- Undo last step of the clustering sequence on each jet breaking each one into two pseudojets



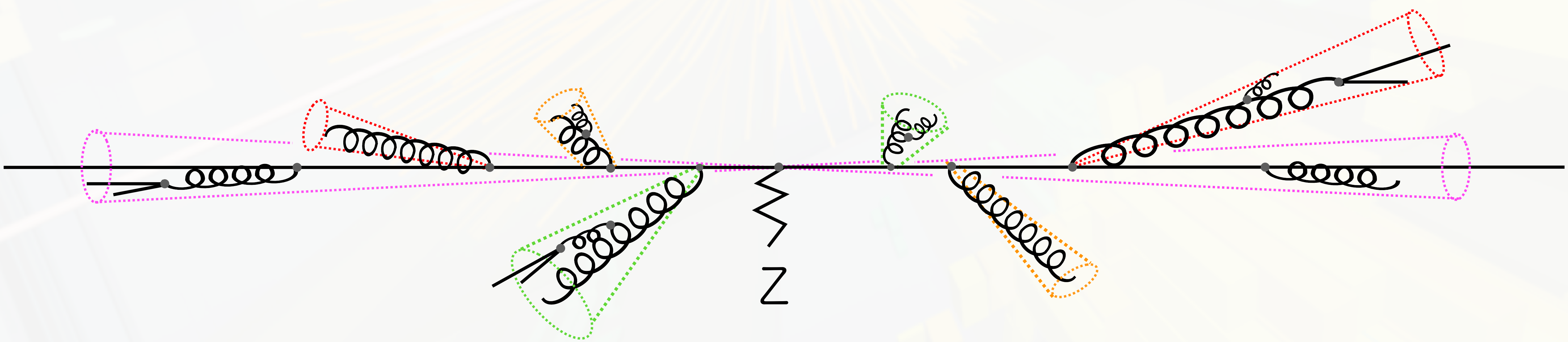
Testing NLL accuracy : multiple emission matrix elements

- Imagine the event is a real event as from mother nature
- Imagine it's clustered into two-jets by the Cambridge algorithm
- Undo last step of the clustering sequence on each jet breaking each one into two pseudojets
- Undo last step of the clustering sequence on each of the hardest pseudo jets just produced



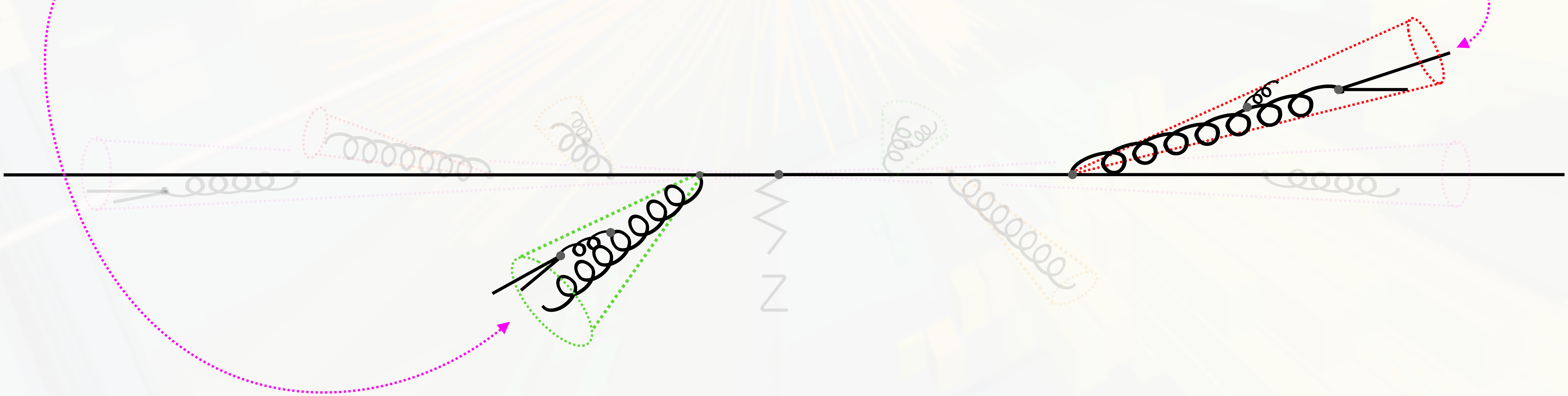
Testing NLL accuracy : multiple emission matrix elements

- Imagine the event is a real event as from mother nature
- Imagine it's clustered into two-jets by the Cambridge algorithm
- Undo last step of the clustering sequence on each jet breaking each one into two pseudojets
- Undo last step of the clustering sequence on each of the hardest pseudo jets just produced
- Iterate undoing the clustering on the hardest produced pseudojet until you can't do it anymore



Testing NLL accuracy : multiple emission matrix elements

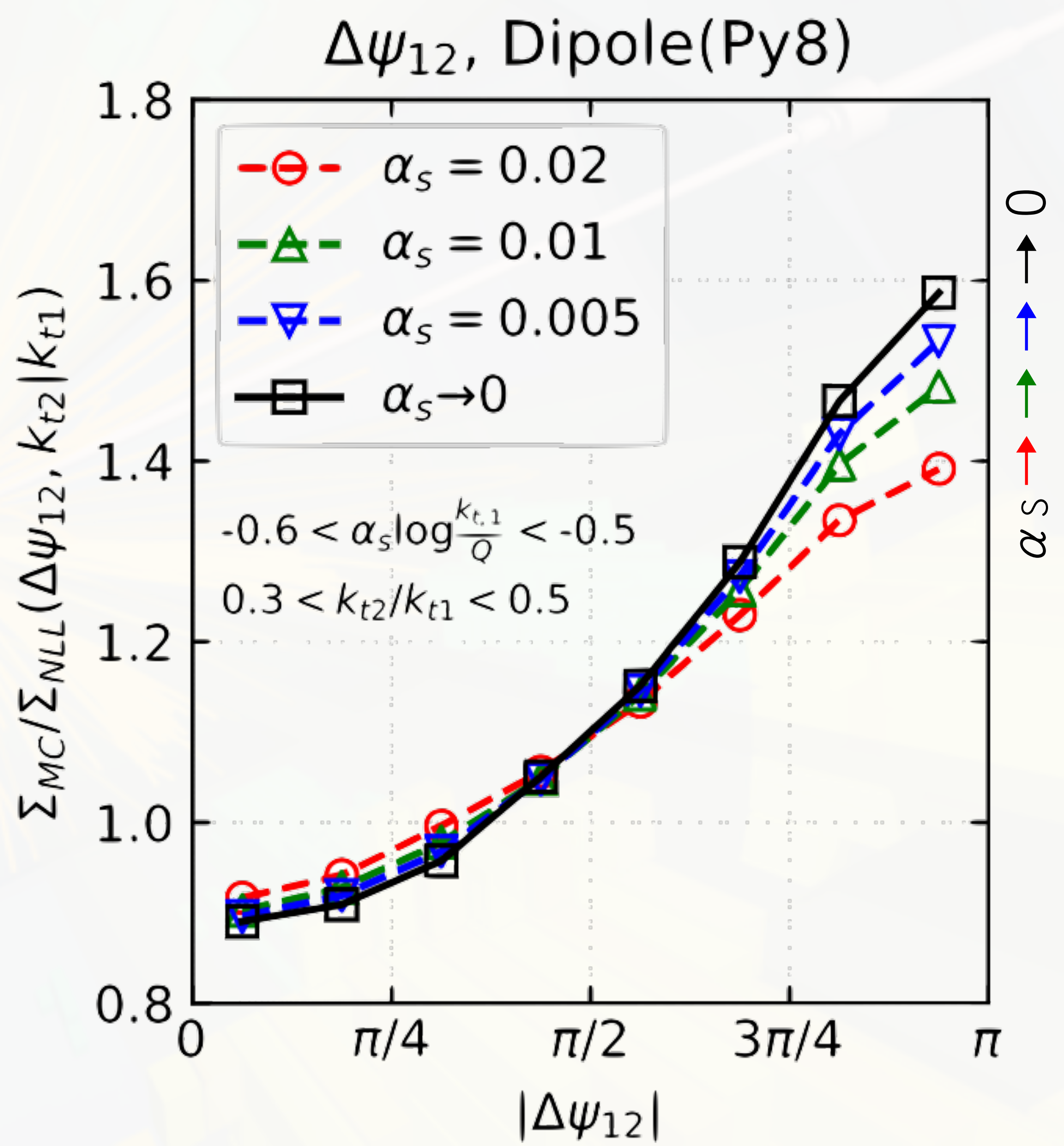
- Imagine the event is a real event as from mother nature
- Imagine it's clustered into two-jets by the Cambridge algorithm
- Undo last step of the clustering sequence on each jet breaking each one into two pseudojets
- Undo last step of the clustering sequence on each of the hardest pseudo jets just produced
- Iterate undoing the clustering on the hardest produced pseudojet until you can't do it anymore
- Now measure the azimuthal sep^{n}, $\Delta\,\psi_{12}$, of the two highest $p_{\perp}$ pseudojets



- These may or may not be in the same jet!

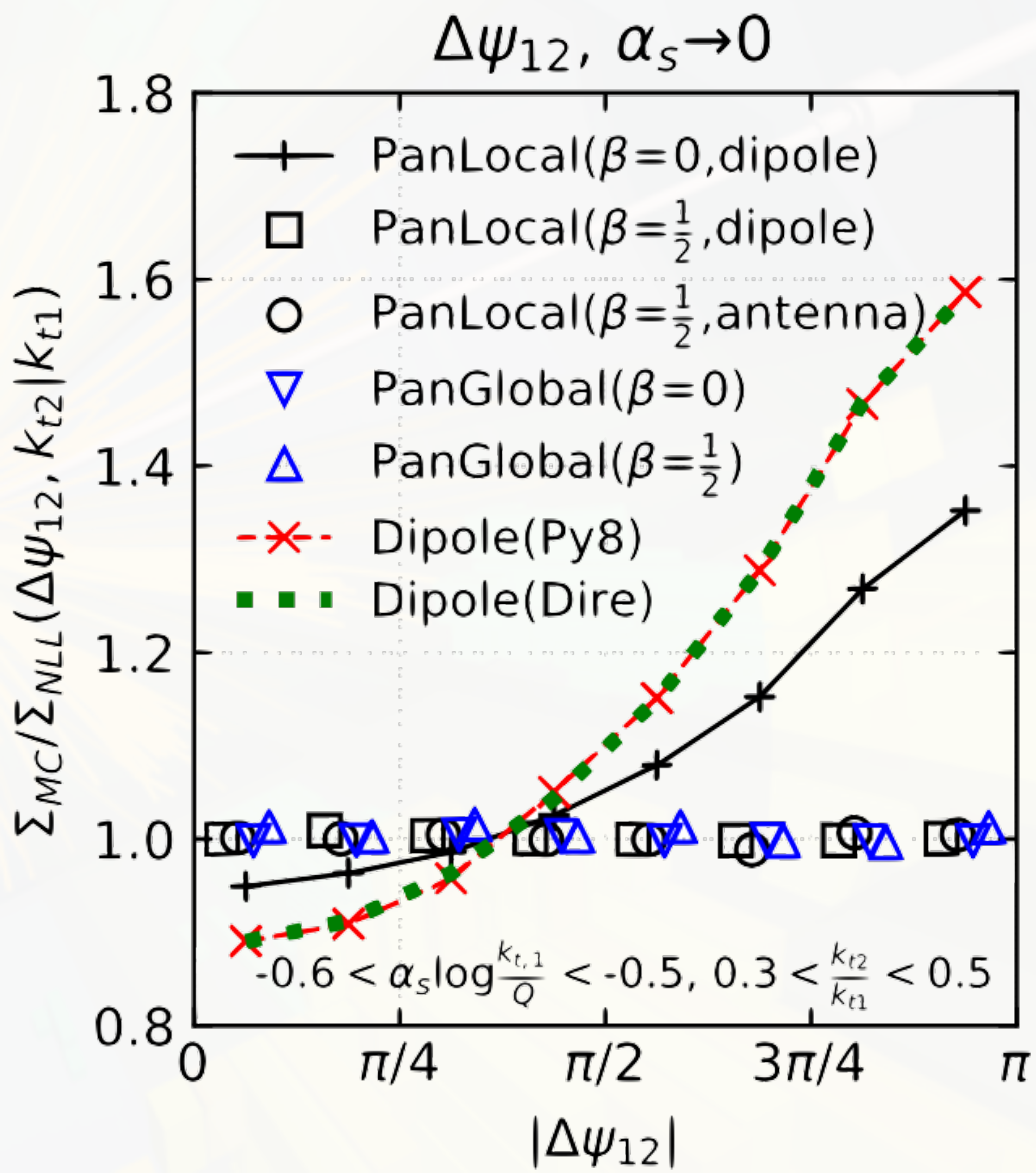
Testing NLL accuracy : multiple emission matrix elements

- We measure the azimuthal sep^n at very small k_t's : $-0.6 < \alpha_S \ln k_{t,1}/Q < -0.5$
- We require the two pseudojets have commensurate k_t's: $0.3 < k_{t,2}/k_{t,1} < 0.5$
- The only logarithms that can develop are therefore due to large angular sep^{n}s
- We know that in this limit of commensurate k_t's and strong A.O. all emissions are blind to each other: $\Delta\psi_{12}$ dist^n is flat for NLL QCD !
- We extrapolate $\Delta\psi_{12}$ dist^n to $\alpha_S \to 0$



Testing NLL accuracy : multiple emission matrix elements

- Exact same pattern of results unfolds as for the global observables [predictably]
- But deviations from NLL are bigger
- PYTHIA8 & DIRE k_t ordered local recoil PSMCs deviate up to 60% from NLL
- Deviation in H→ gg case only goes up to 30% ; additional colour line in case of gluon jets means pseudojets 1 & 2 half as likely to be colour connected
- Such deviatⁿs can bias ML based analysis



Summary

- PSMCs are central everyday tools in the LHC physics programme
- PSMCs are subject to a high-level of validation w.r.t each other & data
- We advocate supplementing this with additional theoretical validation
    i. examining the extent to which they capture singularities of multi-parton MEs
    ii. checking how their predictions compare to logarithmic resummation
- We find existing dipole PSMCs violate colour coherence leading to LL & NLL issues
- We point to key physical elements in NLL resummation of global and non-global observables, as mandatory design constraints on PSMCs
- Concrete local- & global-recoil dipole PSMCs based on these were defined
- Full-fledged implementations were proven to be NLL for a wide range of observables

NLL building blocks : ME for strong energy ordering

- Non-global observables involve non-trivial partitions in phase space, e.g. jet-cone
- Resummation requires correct handling of emissions commensurate in angle, ordered in energy
- $E_{,i+1} \ll E_{,i}$ implies eikonal approximation
- In the large-Nc limit dipoles don't talk to each other [all charged differently]
- Ratio of n+1- to n-particle MEs for a given colour config^{n} is sum of MEs for each dipole to emit
- Summing and symmetrising the latter over all possible orderings gives

$$dP_n \simeq \frac{1}{n!}\prod_{i=1}^{n}\frac{\alpha_s}{\pi}\frac{d\omega_i}{\omega_i}\frac{d^2\Omega}{4\pi}N_c\sum_{\pi_n}\frac{p_1\cdot p_2}{(p_1\cdot k_{i_1})(k_{i_1}\cdot k_{i_2})\ldots(k_{i_n}\cdot p_2)}$$

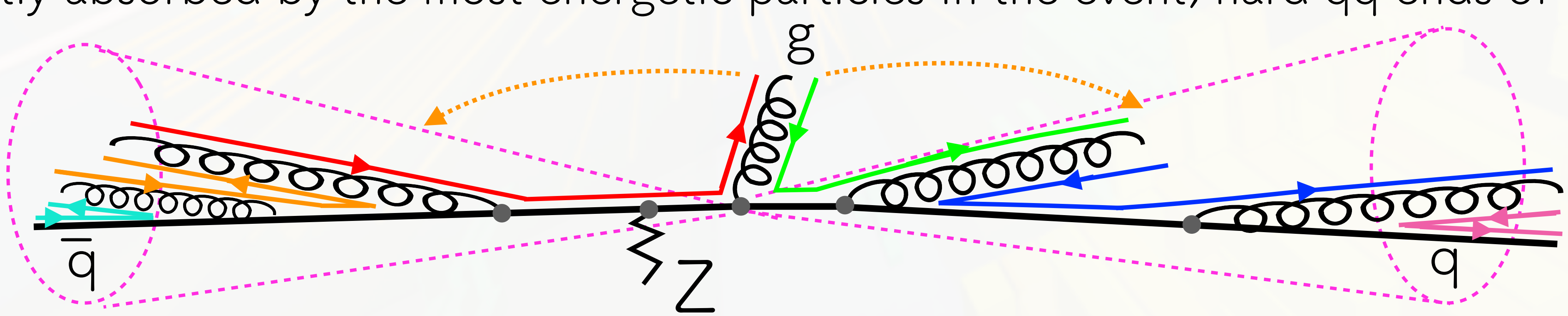
- N.B. unlike the A.O. limit, in the limit $E_{,i+1} \ll E_{,i}$ recoil can be safely forgotten about

NLL PSMC with an event-wide global recoil scheme : PanGlobal

- The longitudinal recoil from each emission is still balanced within the emitting dipole like so

$$\bar{p}_k = a_k \tilde{p}_i + b_k \tilde{p}_j + k_\perp$$
$$\tilde{p}_i \dashrightarrow \bar{p}_i = (1 - a_k)\tilde{p}_i \,,$$
$$\tilde{p}_j \dashrightarrow \bar{p}_j = (1 - b_k)\tilde{p}_j \,.$$

- The event started with momentum Q and now has momentum Q+$k_\perp$
- All particles in the event are rescaled by the same factor, r, to maintain its invariant mass
- $k_\perp$ balanced by boosting the event from r(Q+$k_\perp$) back to Q
- $k_\perp$ dominantly absorbed by the most energetic particles in the event; hard $q\bar{q}$ ends of dipole chain



- Ensures correct pattern of emission for commensurate $k_\perp$ A.O. emissions and commensurate angle energy ordered emissions without changing the ordering variable: $k_\perp$ ordering will work [$0 \leq \beta < 1$]